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THE MATHEMATICAL GAZETTE

EDITED BY
T. A. A. BROADBENT, M.A.

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THE STYLE OF THE *GAZETTE*.

With the beginning of our sixteenth volume, a larger size of type will be used in printing the *Gazette*. Five numbers will be issued during the year, instead of six, but the average yearly content of the *Gazette* will be maintained. The first number in the new style will appear on February 1st, 1932, and succeeding numbers in May, July, October and December.

A complete index to Vols. I.-XV. is in preparation.

A NOTE ON "INERTIA".

BY V. V. NARLIKER, B.A., Isaac Newton Student.

ON page 100 of his *Theory of Sound*, vol. i. (2nd edn.), Rayleigh makes the following remark: "Constraint increases the moment of inertia". Rayleigh probably had in his mind the case of two or three coordinates only. But it appears that the remark is true in a wider sense if another physical quantity is considered in place of the moment of inertia; and the former reduces to the latter when the material system is suitably constrained. This physical quantity will be referred to in this note as the Rayleigh inertia or merely inertia. The following is merely a verification of the remark in this sense.

Consider a natural dynamical system with vis viva

$$2T=\sum_{s=1}^{m}\sum_{r=1}^{m} a_{rs}\dot{q}_r\dot{q}_s \quad (a_{rs}=a_{sr}) \qquad \text{.............................(1)}$$

Let the motion be started by a blow P_1 in the coord. q_1. Writing the Lagrangian equations of motion, we get

$$P_1=\dot{q}_1\frac{\Delta}{A_{11}}, \qquad \text{.....................................(2)}$$

where Δ is the determinant $\| a_{rs} \|$ and A_{11} the cofactor of a_{11}. Following Rayleigh, we will call $\frac{\Delta}{A_{11}}=m_1$ the inertia in the coord. q_1. It is clear that $m_1>0$.

Let the following constraint be supposed to have been introduced in the system before the blow is given:

$$b_{12}\dot{q}_2+b_{13}\dot{q}_3+\dots b_{1n}\dot{q}_n=0, \qquad \text{.....................................(3)}$$
$$b_{1r}=b_{1r}(q_1, q_2, \dots q_n), \quad r=2, 3, \dots n.$$

The coefficient of $\dot{q}_1$ in the last equation is zero, since our object is to see how the inertia in q_1 is affected by restricting the constraint only to some or all of the other coords., so that no impulsive reactions are produced in the coord. q_1 itself. The modified equations of motion may be written as follows:

$$\left.\begin{aligned} p_1&=a_{11}\dot{q}_1+\dots a_{1n}\dot{q}_n, \\ \mu b_{1r}&=a_{r1}\dot{q}_1+\dots a_{rn}\dot{q}_n, \quad r=2, 3, \dots n \end{aligned}\right\} \qquad \text{.....................(4)}$$

From (3) and (4),

$$p_1 = \dot{q}_1(m_1 + \delta m_1), \quad \text{...(5)}$$

where

$$m_1 + \delta m_1 = \frac{b_{12}^2 \dfrac{\partial \Delta}{\partial a_{22}} + 2b_{12}b_{13}\dfrac{\partial \Delta}{\partial a_{23}} + b_{13}^2 \dfrac{\partial \Delta}{\partial a_{33}} + \dots}{b_{12}^2 \dfrac{\partial^2 \Delta}{\partial a_{11}\partial a_{22}} + 2b_{12}b_{13}\dfrac{\partial^2 \Delta}{\partial a_{11}\partial a_{23}} + \dots}. \quad \text{...............(6)}$$

From (2) and (6),

$$\delta m_1 = \frac{\left(b_{12}\dfrac{\partial \Delta}{\partial a_{12}} + b_{13}\dfrac{\partial \Delta}{\partial a_{13}} + \dots + b_{1n}\dfrac{\partial \Delta}{\partial a_{1n}}\right)^2}{\dfrac{\partial \Delta}{\partial a_{11}}\left\{b_{12}^2 \dfrac{\partial^2 \Delta}{\partial a_{11}\partial a_{22}} + 2b_{12}b_{13}\dfrac{\partial^2 \Delta}{\partial a_{12}\partial a_{13}} + \dots\right\}}; \quad \text{......(7)}$$

$$\therefore \ \delta m_1 \geqslant 0.$$

But eliminating the velocities from (3) and (4),

$$p_1\left[b_{12}\frac{\partial \Delta}{\partial a_{12}} + b_{13}\frac{\partial \Delta}{\partial a_{13}} + \dots + b_{1n}\frac{\partial \Delta}{\partial a_{1n}}\right]$$
$$= \pm\mu\left[b_{12}^2\frac{\partial \Delta}{\partial a_{22}} + 2b_{12}b_{13}\frac{\partial \Delta}{\partial a_{23}} + \dots\right]. \quad \text{.......................(8)}$$

Hence as neither $\mu=0$ nor all " b's " equal to zero is possible

$$\delta m_1 \neq 0.$$

It may be remarked that when the coord. q_1 is normal,

$$\frac{\partial \Delta}{\partial a_{1r}} = 0, \quad (r=2, 3, \dots n)$$

and in that case $\mu=0$.

Suppose the constraint is more complicated and is capable of being expressed by a group of linear equations of the form

$$b_{s2}\dot{q}_2 + b_{s3}\dot{q}_3 + \dots b_{sn}\dot{q}_n = 0, \quad \text{...(9)}$$
$$s = 1, 2, 3, \dots k \leqslant n-1.$$

If the k equations (9) are linearly independent we may write the Lagrange equations

$$\left.\begin{array}{r} a_{11}\dot{q}_1 + a_{12}\dot{q}_2 + \dots a_{1n}\dot{q}_n = p_1, \\ a_{r1}\dot{q}_1 + a_{r2}\dot{q}_2 + \dots a_{rn}\dot{q}_n = \mu_1 b_{1r} + \mu_2 b_{2r} + \dots \mu_k b_{kr} \\ r = 2, 3, \dots n \end{array}\right\}. \quad \text{............10}$$

From (9) and (10) eliminating $\dot{q}_2, \dot{q}_3, \dots \dot{q}_n$ and $\mu_1, \mu_2, \dots \mu_k$,

$$p_1 = \dot{q}_1(m_1 + \delta' m_1),$$

where

$$m_1 + \delta' m_1 = \left|\begin{array}{llll} a_{11} & a_{12} \dots a_{1n} & 0 \ \dots \ 0 \\ a_{21} & a_{22} \dots a_{2n} & b_{12} \dots b_{k2} \\ a_{31} & a_{32} \dots a_{3n} & b_{13} \dots b_{k3} \\ \multicolumn{3}{c}{\dotfill} \\ a_{n1} & a_{n2} \dots a_{nn} & b_{1n} \dots b_{kn} \\ & b_{12} \dots b_{1n} & \dots\dots\dots \\ & b_{k2} \dots b_{kn} & \end{array}\right|$$

$\div$ [the cofactor of a_{11} in the last determinant]. ....(11)

To show that $\delta' m_1 > 0$ is purely an algebraic puzzle.

It is easier to show this by successive transformations.

The procedure may briefly be indicated.

Consider the transformation :

$$\left.\begin{aligned} \dot{q}_r^{(1)} &= \dot{q}_r, \quad r=1, 2, \ldots n-1, \\ \dot{q}_n^{(1)} &= b_{12}\dot{q}_2 + b_{13}\dot{q}_3 + \ldots b_{1n}\dot{q}_n, \quad (b_{1n} \neq 0) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(12)$$

$$2T = \sum_{s=1}^{n} \sum_{r=1}^{n} a_{rs}\dot{q}_r\dot{q}_s = \sum_{s=1}^{n} \sum_{r=1}^{n} a_{rs}^{(1)}\dot{q}_r^{(1)}\dot{q}_s^{(1)}.$$

The constraint (3) reduces to $\dot{q}_n^{(1)}=0$. Hence the system thus constrained is given by

$$2T = \sum_{s=1}^{n-1} \sum_{r=1}^{n-1} a_{rs}^{(1)}\dot{q}_r^{(1)}\dot{q}_s^{(1)},$$

$$m_1 + \delta m_1 = \frac{\Delta^{(1)}}{A_{11}^{(1)}}. \quad \ldots\ldots\ldots\ldots(13)$$

where $\Delta^{(1)}$ is the $n-1$ rowed determinant $|a_{rs}^{(1)}|$ and $A_{11}^{(1)}$ the cofactor of $a_{11}^{(1)}$. The values of $m_1+\delta m_1$ as given by (13) and (6) must evidently be identical. Consider next the constraint

$$b_{22}\dot{q}_2 + b_{23}\dot{q}_3 + \ldots b_{2n}\dot{q}_n = 0,$$

i.e.
$$\left(b_{22} - b_{2n}\frac{b_{12}}{b_{1n}}\right)\dot{q}_2^{(1)} + \left(b_{23} - \frac{b_{13}}{b_{1n}}b_{2n}\right)\dot{q}_3^{(1)}$$
$$+ \ldots \dot{q}_{n-1}^{(1)}\left(b_{2,\,n-1} - \frac{b_{1n-1}}{b_{1n}}b_{2n}\right) = 0. \quad \ldots\ldots\ldots\ldots(14)$$

Let us write (14) as

$$b_{12}^{(1)}\dot{q}_2^{(1)} + b_{13}^{(1)}\dot{q}_3^{(1)} + \ldots b_{1n-1}^{(1)}\dot{q}_{n-1}^{(1)} = 0.$$

The effect of this constraint in the system

$$2T = \sum_{s=1}^{n-1} \sum_{r=1}^{n-1} a_{rs}^{(1)}\dot{q}_r^{(1)}\dot{q}_s^{(1)}$$

may be considered by the method already indicated. In this way the increment in m_1 due to all the k constraints, taken together, may be calculated. Even if the calculations may be laborious, the point is clear that $\delta' m_1 > 0$.

Lastly, I should like to express my thanks to Mr. A. S. Ramsay for pointing out a mistake in my algebra. V. V. N.

GLEANINGS FAR AND NEAR.

824. " And if the Greek Muses wer a graceful company
yet hav we two, that in maturity transcend
the promise of their baby-prattle in Time's cradle,
Musick and Mathematick: coud their wet-nurses
but see these foster-children upgrown in full stature,
Pythagoras would marvel and Athena rejoice."

—Robert Bridges, *The Testament of Beauty*, I. 737
[*Per* Miss M. O. Stephens].

825. Among the lions " were . . . and Davies Giddy, whose face ought to be perpetuated in marble for the honour of mathematics."

—Southey to Coleridge, 1804. Quoted from *Athenaeum*, Jan. 1850, p. 67.

[Davies Giddy assumed his wife's name of Gilbert. He was President of the Royal Society in 1827-1830.]

MATHEMATICS AT THE BRITISH ASSOCIATION, 1931.

Interest for mathematicians in general rather than concern with problems intelligible to small groups was the characteristic of most of the papers in mathematics at the centenary meeting of the British Association. The Department had a full programme for four mornings, and Prof. A. R. Forsyth occupied the chair throughout.

The proceedings began with two model contributions. A polyhedron may have all its faces congruent while the arrangement round one corner is different from that round another; reciprocally, the corners may be indistinguishable while the faces are of several shapes; Prof. D. M. Y. Sommerville exhibited specimens of polyhedra of these two kinds. Mr. H. S. M. Coxeter described a uniform four-dimensional polytope which he discovered recently, pointing out its analogy with the snub cube; he explained how the figure can be derived from more familiar polytopes, and showed a beautiful polyhedron constructed many years ago by Mrs. Boole Stott which is in fact a central "solid" section of it. Some fascinating topological problems were outlined by Mr. M. H. A. Newman, who succeeded in making clear just how a highly abstract theorem on the existence of self-correlated points is relevant to the logical analysis of congruence. At the opposite pole of geometry was an account by Prof. A. J. McConnell of the most recent illustrations of the advantage of expressing dynamical theorems in the language of differential geometry; the solution of a problem in rational mechanics may be made to depend on the construction of a Riemannian space to contain a surface with such a line element that it can not exist in ordinary space; if the null geodesics of the Riemannian space can be found, nothing remains to be done.

Mathematicians no longer in touch with current analysis have heard with curiosity that there are functions described as almost-periodic, and have noticed that these functions formed the subject for which the Adams Prize was awarded this year to Dr. A. S. Besicovitch; a lucid explanation of their nature by Mr. E. H. Linfoot was much appreciated, and it is to be regretted that more time was not allotted to this able expositor. The theory of partitions has developed greatly of recent years; Mr. T. W. Chaundy and Mr. E. M. Wright dealt with the enumeration not of simple partitions but of partitions into parts which are themselves broken up; it is remarkable that the elaboration of the analysis is less formidable than the elaboration of the concept might lead one to fear, and while some light is thrown on the simpler analysis, it becomes evident also that the extensions are far less artificial than they seem at first glance. Other papers on analysis were given by Miss M. L. Cartwright, Mr. R. E. A. C. Paley, and Mr. J. M. Whittaker. Prof. H. W. Turnbull in speaking of matrices had some effective illustrations of the use that can occasionally be made of the trivial cases of a general formula, Mr. T. Smith discussed matrices of a peculiar kind, and Prof. A. R. Richardson gave a lucid and complete account of recent developments of non-commutative algebra.

Mr. J. B. S. Haldane attracted a large audience for a paper on the difference equations embodying the mathematical formulation of certain biological problems; a simple question on the distribution of inherited qualities may lead to a set of equations too numerous or too complicated in form for solution in detail, but the biologist's chief concern is with the asymptotic character of the solution, and this can sometimes be learnt; if the investigation leads to topological questions regarding space of four or more dimensions, the welfare of the race depends more than is commonly suspected on the encouragement given to the study of the most intangible of all branches of mathematics. Dr. G. Temple's explanation in general terms of the conflicting difficulties which a satisfactory wave equation must overcome was equally isolated from the rest of the papers, but was no less appreciated for that reason.

A morning was devoted to fluid motion, but the contributions were somewhat heterogeneous. Prof. G. I. Taylor spoke of turbulence and stability in a fluid of variable density, making a comparison between theoretical results and observations taken in certain fjords where sea water and fresh water mingle. Prof. R. V. Southwell outlined a method by which he and Mr. H. B. Squire are approaching solutions of the equation governing viscous flow round an obstacle; the first approximation depends on an equation which is formally the same as that used by Oseen for the same purpose, but the variables and constants are adjusted to the individual question by a conformal transformation; there is a prospect of further improvement by superposition, but as yet the details of the elementary solutions are not all worked out. Dr. S. Goldstein's account of his investigation of motion between rotating cylinders caused Prof. Taylor, who was the first many years ago to tackle the simplest form of this problem successfully, to feel, he said, like a very small father confronted with an enormous son, and caused Prof. Forsyth to express alarm lest the handling of Fourier series was as reckless as the summary inevitably made it appear. A communication was received from Prof. A. Rosenblatt, and after Prof. J. R. Partington had described some experiments of his and Mr. N. L. Anfilogoff's on flow through curved pipes, it was a great pleasure to hear Sir Horace Lamb commenting on current problems in a branch of mathematics with which his name has been associated for half a century.

The subject of interpolation is timely for the British Association, which has a volume of tables in the press. Dr. J. Wishart dealt at length with the variety of considerations which a compiler, restricted perhaps in time and money, must balance, and discussed in detail a formula published by C. Jordan in 1928 for interpolating from the tabular entries alone, without differencing; this formula relieves the compiler, and the burden of labour thrown on the user if heavier than is involved in the use of Everett's formula when differences are provided is so much lighter than he must carry if he computes his own differences that for a table of which only occasional use is anticipated the omission of all aids to interpolation may sometimes be defended. This paper also found an expert critic, for Dr. W. F. Sheppard was there to maintain interest in the matter.

Speculative astronomy, numerical calculation, and functional analysis met in a series of papers on Emden's equation, the second-order differential equation on which the credibility of certain types of stellar models depends. Mr. D. H. Sadler who has been engaged on computation in collaboration with Dr. L. J. Comrie, and Mr. N. Fairclough who has been associated with Prof. E. A. Milne, stated the ranges covered by their work, the methods used, and the precautions taken. In Prof. Milne's absence, his account of the way in which the tabulation of certain functions of solutions rather than of the solutions themselves simplifies procedure and expedites applications was presented by Mr. R. H. Fowler, who in the last paper of the meeting described his own method of analysing the singularity of any real solution at the origin by using as a mesh of reference curves the particular solutions discovered by Emden for which there is no singularity there; it was a disappointment to learn that in Mr. Fowler's opinion this extraordinarily beautiful device depends for its success on very special properties of the equation and is not likely to prove of wider utility in the study of second-order equations.

Abstracts and a bibliography will as usual appear in due course in the Association's *Report* of the meeting. E. H. N.

---

826. This morning Solon, Lycurgus, Demosthenes, Archimedes, Sir Isaac Newton, Lord Chesterfield and a great many more went away in one post-chaise.—Samuel Rogers, after seeing Brougham off from Panshanger.

THE POLAR EQUATIONS OF A CURVE.

By C. Fox.

It is well known that a curve has only one equation in Cartesian co-ordinates when referred to a given pair of lines as axes, but it does not appear to be so well known that a curve can have many equations in polar co-ordinates, all referred to the same pole and initial line. This fact, although of considerable importance in the study of curves by means of their polar equations, appears to be consistently ignored by most text-books.

The straight line has only one equation in polars, the conic section has two, and there are curves, for example the equiangular spiral, which have an infinite number of polar equations. The theory is as follows:

Let (r, θ) be the polar co-ordinates of any point P referred to a given initial line and pole, then for all positive or negative integral values of n (including $n=0$) the number-pairs

$$(r, 2n\pi+\theta), \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$(-r, \overline{2n+1}\pi+\theta), \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

are also co-ordinates of P referred to the same initial line and pole. Hence if P lies on the curve

$$f(r, \theta)=0, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

since (1) and (2) are also co-ordinates of P, it follows that

$$f(r, 2n\pi+\theta)=0, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$f(-r, \overline{2n+1}\pi+\theta)=0, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

for all integral values of n, zero included. Thus the curve on which P lies has equations given by (4) and (5) as well as that given by (3), and it may happen that the form of (4) and (5) differs from that of (3). It may appear superfluous but it is well worth while pointing out that the equations

$$f(r, \theta)=f(r, 2n\pi+\theta)=f(-r, \overline{2n+1}\pi+\theta)$$

have *not* been proved. What has been proved is that if (r, θ) satisfies either (4) or (5) then the point (r, θ) lies on the curve (3) although it may not satisfy (3).

I now proceed to deal with some special cases. The equation of a straight line in polars is of the form

$$A/r=B\cos\theta+C\sin\theta. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

On substituting either (1) or (2) in (6) we are led back to (6). Hence a straight line has only one polar equation.

The equation of a conic referred to a focus as pole and its major axis as initial line is in the usual notation,

$$l/r=1+e\cos\theta. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

If we substitute (1) in (7) we are led back to (7), but if we substitute (2) in (7) we have

$$l/r=-1+e\cos\theta. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Hence if (r, θ) satisfies either (7) or (8) it lies on the conic (7). Hence a conic section has two polar equations.

Evidently if (r, θ) satisfies one of the equations (7) or (8) it cannot satisfy the other.

This result is of considerable importance in the study of conic sections by means of their polar equations. For example, suppose we wish to find the points of intersection of the two conics whose equations are

$$l/r = 1 + e\cos\theta, \qquad (9)$$

$$l'/r = 1 + e'\cos\theta. \qquad (10)$$

On eliminating r we have

$$\cos\theta = (l - l')/(el' - e'l). \qquad (11)$$

This equation for θ has only two roots lying between 0 and 2π. Hence equations (9) and (10) give us at most two of the four intersections of the conics whose equations are (9) and (10). We cannot obtain the other two points of intersection unless we use the equation of one of the conics in the form given by (8). The other two points of intersection are then given by eliminating r from

$$l/r = -1 + e\cos\theta, \qquad (12)$$

$$l/r = 1 + e'\cos\theta, \qquad (13)$$

leading to

$$\cos\theta = (l + l')/(el' - e'l). \qquad (14)$$

Suppose that we use the equations of the conics in the form

$$l/r = -1 + e\cos\theta, \qquad (15)$$

$$l'/r = -1 + e'\cos\theta. \qquad (16)$$

On eliminating r we have

$$\cos\theta = (l' - l)/(el' - e'l). \qquad (17)$$

As this equation differs from both (11) and (14) it might appear that there are more than four points of intersection. It is easily shown, however, that this is not so. For let A be a point of intersection of the two conics given by equations (9) and (10) and let the co-ordinates of A be (a, α). Then

$$l/a = 1 + e\cos\alpha,$$

and α satisfies equation (11).

Since α satisfies (11) it follows that $\pi + \alpha$ satisfies (17), and so, substituting $\pi + \alpha$ for θ in (15), we find that the corresponding value of r is $-a$. Hence a point of intersection of the two conics when equations (15) and (16) are used is $(-a, \pi + \alpha)$, which is the point A.

It is now evident that the two points of intersection given by (15) and (16) are the same as those given by (9) and (10).

Similarly, the two points of intersection given by

$$l/r = 1 + e\cos\theta, \qquad (18)$$

$$l'/r = -1 + e'\cos\theta, \qquad (19)$$

are the same as those given by (12) and (13).

I conclude by showing that the equiangular spiral

$$r = ae^{\theta\cot\alpha} \qquad (20)$$

has an infinite number of equations in polar co-ordinates. On substituting from (1) and (2) in (20), we have

$$r = ae^{2n\pi\cot\alpha}e^{\theta\cot\alpha} \qquad (21)$$

$$-r = ae^{(2n+1)\pi\cot\alpha}e^{\theta\cot\alpha}, \qquad (22)$$

where n is any portion or negative integer (zero included). Hence if (r, θ) satisfies any of the equations (21) or (22) it lies on the equiangular spiral (20), although it does not satisfy equation (20) unless it satisfies (21) with the value $n = 0$. C. F.

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827. Oct. 8, 1830. Galileo was a great genius, and so was Newton; but it would take two or three Galileos and Newtons to make one Kepler.—S. T. Coleridge, *Table Talk and Omniana*.

HOW OUGHT A LOGARITHM TO BE DEFINED?

By R. F. Muirhead, D.Sc.

Mr. Fletcher begins his article in the January number of the *Gazette* by giving two reasons why a logarithm ought not to be defined as the index of the power of the base which is equal to the number. But are these reasons well-founded? One is that it does not afford a basis for the more advanced parts of the theory of logarithms. That statement would require a proof that might be very difficult or perhaps impossible to produce.

The other reason given is that the definition criticised does not show how a logarithm is to be calculated. Now several good methods have been devised for calculating logarithms from the direct use of this definition. Some little time ago a note of mine in the *Gazette* * called attention to what is perhaps the simplest and most direct of these methods. As it does not appear to be well known, the following example of its application may be of interest.

It is required to calculate the first three digits of the mantissa of the logarithm of 234, that is, of the logarithm of 2·34. The working, using abbreviated multiplication, is as follows:

```
    2 3 4
   ---------
    4 6 8
      7 0 2
        9 3 6
    5 4 7 6   ... ·5 × 0.
   ---------
  2 7 3 8 0
  2 1 9 0
    3 8 3
  ---------
  2 9 9 5     ... ·25 × 1.
  5 9 9 0
  2 6 9 5
    2 7 0
      1 5
  ---------
  8 9 7 0     ... ·125 × 0.
 ---------
 7 1 7 6 0
   8 0 7 3
   6 2 8
 ---------
 8 0 4 6      ... ·0625 × 1.
 ---------
 6 4 3 6 8
   3 2 2
     4 8
 ---------
 6 4 7 4      ... ·03125 × 1.
```

```
    6 4 7 4
   ---------
    3 8 6 4
      2 5 9
        4 5
    4 1 7     ... ·015625 × 1.
   ------
  1 6 7
      4
      3
  ------
  1 7 4       ... ·0078125 × 1.
  1 7 4
  1 2 2
      7
  ------
  3 0 1       ... ·00390625 × 0.
  ------
    9 0       ... ·00195 × 0.
  ------
  8 1 0       ... ·00097 × 1.
  ------
    6 5
    ·1
  ------
    6 6       ... ·00048 × 1.
  ------
  3 9
    4
  ------
  4 3         ... ·00024 × 1.
  ------
```

Adding up the numbers in italics, we get $\log 2{\cdot}34 = {\cdot}369$. To explain the method: suppose $1 < a < 10$.

* *Math. Gazette*, xi, p. 389 (1923).

Then: $a^2 = 10^{\alpha} \times b,\quad 1 < b < 10,\quad \alpha = 0$ or 1.
$b^2 = 10^{\beta} \times c,\quad 1 < c < 10,\quad \beta = 0$ or 1.
$c^2 = 10^{\gamma} \times d,\quad 1 < d < 10,\quad \gamma = 0$ or 1.

This process of repeated squaring is to be continued as far as required. If we stop at d, we have

$$a = 10^{\cdot 5\alpha + \cdot 25\beta + \cdot 125\gamma} \times d^{\cdot 125};$$

$$\log_{10} a = \cdot 5\alpha + \cdot 25\beta + \cdot 125\gamma + \cdot 125 \log_{10} d.$$

Then since the last term is less than ·125, we obtain, by three squarings, the value of $\log_{10} a$ with a possible error of at most ·125.

In the numerical example given above

$$a = 2\cdot 34,\ \alpha = 0;\quad b = 5\cdot 476,\ \beta = 1;\quad c = 2\cdot 995,\ \gamma = 0;\ \ldots$$

and the process is continued to twelve squarings, so that the difference between the sum of the numbers in italics and log 2·34 is ·00024 × log 4·3 ..., and is less than ·00024, and therefore does not affect the third digit of the mantissa.

Of course, if a more exact result is wanted, more squaring is necessary. To get the mantissa to n decimal places, $10n/3$ squarings will suffice, save in very exceptional cases, as when the last digit to be calculated is followed by 4999 ... or 5000 ... . By $10n/3$ squarings we always get a result in which the nth digit does not differ from its true value by more than unity.

With a little practice it is easy to find the logarithm of any number to four decimal places within less than half an hour.

Note that in the contracted work, since all the multiplications are squarings, it has not been necessary to write the multiplier separately. Further economies in the calculation can often be effected. Thus, the figures in the result of the eighth squaring in the above example, namely 301, are so near the value 2995 in the result of the second squaring, that the coefficients immediately following, namely 0, 1, 1, 1, will obviously be repeated after 301 so that eight squarings suffice. R. F. M.

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828. The Admiral (Lord Hood) pointed out to my father a proper person, who having been employed at the royal observatory at Greenwich, to inspect and correct the calculations that were made for nautical almanacks, has just retired from public life, and settled at Portsmouth with his family, where, desirous of taking three or four young gentlemen intended for the navy, he purposed to devote his time to instruct and lead them through a complete course of mathematics and astronomy. To this person he recommended my being sent for a year; accordingly I was sent to Portsmouth, and being his only pupil, he led me progressively on in a course of study, grounding me well in spherical trigonometry, to which he principally kept me. In hours not assigned for application, he would open to me other branches of calculation and science, which, though not immediately necessary to a sea officer, would occasionally be of infinite use to me in various ways.

During the year I remained with this philosopher, his whole self seemed wrapt in calculation, and his only recreation or cessation therefrom was a couple of hours daily; these he passed in a garret with some canary birds, in which he took great delight; it was his custom to sit and watch them, clean their cage, and give them food, for to me it seemed nothing more. I often laughed thereat, and do now laugh, as he was a married man, and it seemed wonderful to me how the deuce he could find time to court, he being so enamoured with them, the stars, and calculations; I should like to have seen if whilst thus inflated, he made his approaches by spherical movements, for certainly when he was gay, that is gay for a philosopher, he had a peculiar jerk in his walk, and when he turned about, it gave the flap of his coat a sort of motion that put me always in mind of a spherical triangle.—Rear-Admiral Raigersfeld, *The Life of a Sea Officer* (c. 1830), Reprint, 1929, pp. 37-38. [Per Mr. Puryer White.]

NOTE ON SOLVING ALGEBRAIC EQUATIONS BY ROOT-CUBING.

By A. C. Aitken, D.Sc.

§ 1. The advent of the modern calculating machine has reduced the labour of computation in an extraordinary degree; continued products, long division, and so on, offer no terrors to anyone who has access to any of the latest excellent machines; even the extraction of a square root can be performed to ten digits in a minute or two. Under these conditions the numerical solution of algebraic equations, a process stigmatised * in the seventeenth century as intolerably laborious, can even be in danger of becoming a vice: the present note owes its origin to a mild indulgence.

§ 2. The admirable extension made some years ago by Brodetsky and Smeal † to Graeffe's root-squaring method of solving algebraic equations is so simple and efficient that, for evaluating complex roots at any rate, nothing more adapted to computation by machine could be desired. As far as equations with real roots are concerned, the ordinary unextended root-squaring method is very powerful, perhaps its only disadvantage, and this in many cases an unimportant one, being that the sign of the roots has to be fixed afterwards by graphical or other considerations. A method of root-cubing would be free from ambiguity of sign, and it is demonstrated below that we can root-cube very simply in two stages, both being of the kind suited to modern machines, namely the formation of binary products and the subsequent combination of these by additions or subtractions.

§ 3. Let $$a_0x^n - a_1x^{n-1} + a_2x^{n-2} - \dots + (-)^n a_n = 0 \quad \dots\dots\dots\dots(1)$$

be an equation having roots $x_1, x_2, x_3, \dots, x_n$ in descending order of absolute value. The "Encke equation" (*i.e.* that which has the roots of (1) with sign changed) is

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0. \quad \dots\dots\dots\dots(2)$$

Also, ω and ω^2 being the complex cube roots of unity, the equations having roots $-\omega x_r$ and $-\omega^2 x_r$ respectively are

$$a_0x^n + \omega a_1x^{n-1} + \omega^2 a_2x^{n-2} + a_3x^{n-3} + \dots = 0, \quad \dots\dots\dots\dots(3)$$

and

$$a_0x^n + \omega^2 a_1x^{n-1} + \omega a_2x^{n-2} + a_3x^{n-3} + \dots = 0. \quad \dots\dots\dots\dots(4)$$

The continued product of (2), (3), (4) and writing of z for x^3 in the result would give us an equation in z having roots $-x_1^3, -x_2^3, \dots$. Multiplying together first (3) and (4) we obtain, as coefficients of powers of x from x^{2n} downwards,

$$a_0^2, \; -a_0a_1, \; a_1^2 - a_0a_2, \; -a_1a_2 + 2a_0a_3, \; a_2^2 - a_1a_3 - a_0a_4, \; -a_2a_3 + 2a_1a_4 - a_0a_5, \dots .$$

The rule is evident and simple: *Squared a's receive +; products of different a's receive −, unless their suffixes differ by a multiple of 3, when they receive + and are doubled.* The new coefficients are isobaric in the a's, and are easily run off on the machine.

If we call these coefficients, $2n+1$ in number,

$$b_0, \; b_1, \; b_2, \dots, b_{2n},$$

the second step, simple multiplication by (2), will result in the set of coefficients, now $n+1$ in number, of powers of z,

$$a_0b_0, \quad a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0, \quad a_0b_6 + a_1b_5 + \dots + a_6b_0, \dots .$$

* Whittaker and Robinson, *Calculus of Observations*, p. 79.

† *Proc. Camb. Phil. Soc.* 22 (1923), pp. 83-87.

These are therefore the coefficients in the equation having for roots the negative cubes of the roots of (1). The similarity to the ordinary root-squaring process cannot fail to be observed, and the theory of root-approximation is the same.

We have now root-cubed once. The process is then iterated until the coefficients show, just as in the root-squaring process, that the roots have been sufficiently separated, when the solution is completed by logarithms. In the later stages of the process the dominant terms of the coefficients obtained are respectively

$$a_0 b_0, \quad a_1 b_2, \quad a_2 b_4, \quad a_3 b_6, \ldots,$$

and the process is stopped when these dominate the rest, to the number of significant digits desired.

In the scheme of computation illustrated below these b's come exactly below the corresponding a's, and it is advantageous to calculate the dominant central terms $a_r b_{2r}$ first.

Example. To four places of decimals, solve *

$$x^3 + 9x^2 + 23x + 14 = 0.$$

[*N.B.*—We shall represent, where necessary, *e.g.* 1234 by 4·1234, ·00123 by $\bar{2}$·123, and so on. Thus the characteristic prefix gives the number of *significant digits preceding*, or, if negative, the number of *ciphers following* the decimal point of the number represented. If this characteristic be set to the pointer of a machine there is never any trouble about the order of the answer.]

x	1		−9		23		−14	(*a*)
	1	9	58	179	403	322	196	(*b*)
x^3	1		−150		4061		−2744	
	1	150	18439	6·602366	8·16080	8·11143	7·75295	
x^9	1		− 7·15558		11·61981		−11·20661	
	1	7·15558	13·23585	17·96430	22·38416	22·12806	21·42688	
x^{27}	1		−19·34765		33·23814		−31·88198	
	1	19·34765	38·12086	51·82775	65·56692	64·21000	62·77789	
x^{81}	1		−56·42105		98·13498		−93·68608	

[For example, the 403 of the second row is obtained by $23^2 - (-9)(-14)$; the 179 of the second row by $(-9)(23) + 2(1)(-14)$; and the 4061 of the third row by $(403)(23) - (14)(179) - (9)(322) + (1)(196)$.]

Hence, by division of consecutive entries in the last row,

$$x_1^{81} = -\cdot 42105 \times 10^{56}; \quad x_2^{81} = -\cdot 32127 \times 10^{42}; \quad x_3^{81} = -\cdot 50828 \times 10^{-4}.$$

Thus $\quad \log|x_1| = \cdot 68671; \ \log|x_2| = \cdot 51243; \ \log|x_3| = \bar{1}\cdot 94699.$

$$x_1 = -4\cdot 8608, \quad x_2 = -3\cdot 2541, \quad x_3 = -0\cdot 8851.$$

§ 4. Where the roots are real, as in our example above, and their signs are not difficult to determine, by inspection or from a graph or otherwise, no advantage is claimed for root-cubing over root-squaring, the latter method being simple, rapid and uniform. Where, however, the question of sign is likely to be troublesome, it is suggested that root-cubing may be of use, and can be kept in reserve. For the purpose of separating the roots it is fairly powerful; thus five root-cubings are equivalent to raising the roots to the 243rd power.

A. C. Aitken.

* Whittaker and Robinson, p. 109.

SOME THEOREMS ON THE INTERSECTIONS OF A CONIC WITH CONCENTRIC CIRCLES.

By W. F. Beard, M.A., M.Sc.

Most of the following theorems regarding the hyperbola of Apollonius and the parabola of Chasles are classical, but not all of them are as well known as they deserve to be, and it is hoped that this purely geometrical treatment may be of interest.

1. If with a fixed point as centre any circle is drawn to cut a central conic in four points, the diagonal points of this quadrangle lie on a rectangular hyperbola through the centres of the conic and circle and with its asymptotes parallel to the axes of the conic.

2. The feet of the four normals from the fixed point to the conic lie on this hyperbola.

3. The centre-locus of the pencil of conics circumscribing any quadrangle defined as in Th. 1 is this same rectangular hyperbola.

4. The sides of the diagonal-point triangles of quadrangles defined as in Th. 1 envelop a fixed parabola which touches the axes of the conic.

5. This parabola touches the tangents to the conic at the feet of the normals to the conic from the fixed point.

6. To find the focus and directrix of this parabola.

7. This parabola is the same for all conics confocal with the given conic.

8. The circumcircles of the diagonal-point triangles of quadrangles defined as in Th. 1 are coaxal.

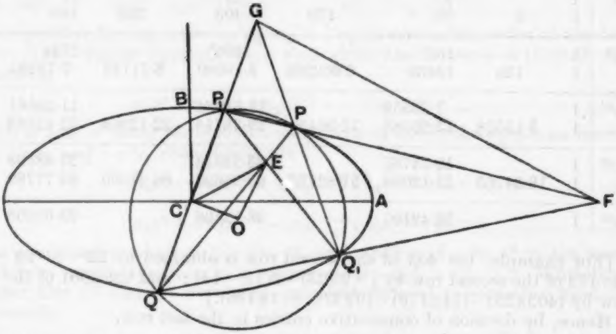

Fig. 1.

1. Let Σ be a conic (see Fig. 1) with semi-axes CA, CB.

Draw any circle with a fixed point O as centre to cut Σ in PP_1QQ_1. Let EFG be the diagonal-points of PP_1QQ_1. Join CE, OE.

EFG is a self-polar triangle for both Σ and the circle.

Thus OE is perpendicular to FG, and CE is conjugate to FG for Σ.

Hence if OE is drawn in any direction, CE is conjugate to lines perpendicular to OE. Thus there is a (1, 1) correspondence between CE and OE.

$\therefore$ the locus of E is a conic through C, O, and similarly for F, G.

If OE is drawn parallel to CB, FG must be parallel to CA and CE must be along CB. Thus the point at infinity on CB is a position of E.

Thus Σ_1, the locus of E, has one asymptote parallel to CB and similarly the other asymptote is parallel to CA.

Thus Σ_1 is a rectangular hyperbola through C, O, with its asymptotes parallel to CA, CB.

2. If the circle centre O is drawn to touch Σ at P, as in Fig. 2, and to cut it in QQ_1, then P_1 coincides with P, so do E, G, and the tangent at P to Σ meets QQ_1 in F. OP is a normal to Σ. Thus P lies on Σ_1, and so similarly do the feet of the other normals from O to Σ.

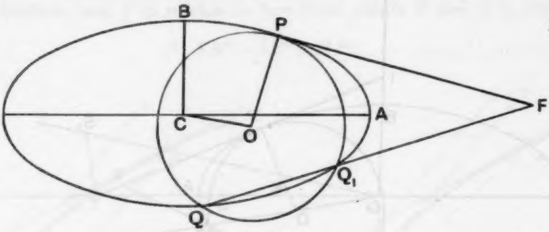

FIG. 2.

COROLLARY. F is a point on Σ_1. Also EG in Fig. 1 is perpendicular to OF.

$\therefore$ in Fig. 2 the tangent at P to Σ_1 is perpendicular to OF and is the polar of F for Σ and the circle.

3. In Fig. 1 the centre locus of conics circumscribing PP_1QQ_1 goes through E, F, G, the diagonal-points of PP_1QQ_1, and C, O are obviously points on it, therefore the centre-locus is Σ_1.

4. Reciprocate the conic Σ with respect to itself (see Fig. 1).

E, F, G reciprocate into FG, GE, EF respectively.

Thus the envelope of FG, GE, EF is a conic Σ_2, the reciprocal of Σ_1 for Σ. Σ_1 goes through C; $\therefore$ Σ_2 touches the line at infinity and is a parabola.

The point at infinity on CA lies on Σ_1; $\therefore$ CB touches Σ_2.

Thus FG, GE, EF envelop a parabola which touches CB and similarly CA.

5. When the circle, centre O, touches Σ at P, EF in Fig. 1 becomes the tangent at P to Σ in Fig. 2. Thus the tangents to Σ at the feet of the normals from O touch Σ_2.

COROLLARY. In the Corollary to Th. 2 it was proved that the tangent at P to the rectangular hyperbola Σ_1 was the polar of F for the circle, and this is also the polar of F for Σ. Hence F in Fig. 2 lies on the parabola Σ_2 and PF is the tangent at F. Further, it was proved that F was on Σ_1; $\therefore$ the polar of F for Σ touches Σ_2.

Another tangent to Σ_2 is the polar of O for Σ.

6. As regards the parabola Σ_2, we shall prove (1) that CO is the directrix, (2) that, if S_2 is its focus and S is either focus of Σ, then $S_2\hat{C}S = O\hat{C}S$ and $CS_2 \,.\, CO = CS^2$.

Let the tangent at P in Fig. 2 meet CA, CB in T, t and let OP meet CA at K. Join S_2T.

(1) CA, CB are perpendicular tangents to Σ_2; $\therefore$ C is on the directrix.

Also in Fig. 1, O is the orthocentre of the triangle EFG which is a tangent triangle to Σ_2, $\therefore$ O is on the directrix. Thus CO is the directrix of Σ_2.

(2) See Fig. 3. Since S_2 is the focus and CO the directrix of Σ_2, CS_2, CO are conjugate lines for Σ_2 and CA, CB are tangents.

$\therefore$ $C\{BS_2AO\}$ is harmonic and $A\hat{C}B = 90°$. $\qquad\therefore$ $S_2\hat{C}S = O\hat{C}S$.

In Th. 5, tPT, the tangent at P to Σ, was proved to be the tangent at F to Σ_2; thus tCT is a tangent triangle to Σ_2, $\therefore$ S_2 lies on the circle tCT.

$\therefore$ $C\hat{S_2}T = C\hat{t}T = C\hat{K}O$. $\because$ $t\hat{C}K = 90° = t\hat{P}K$.

Thus in the triangles CKO, CS_2T, $C\hat{K}O = C\hat{S}_2T$ and $O\hat{C}K = T\hat{C}S_2$; $\therefore$ these triangles are similar; $\therefore \dfrac{CS_2}{CT} = \dfrac{CK}{CO}$; $\therefore CS_2 . CO = CK . CT = CS^2$, $\because PK$, PT are the tangent and normal at P to Σ.

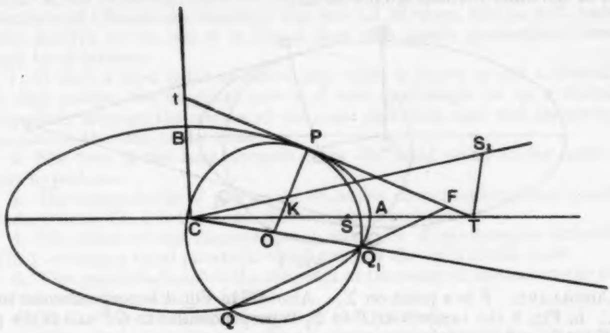

FIG. 3.

7. From Th. 6 it follows that, if C, S, and O are fixed, so is S_2. Thus Σ_2 is the same for any conic confocal with Σ.

It may also be noted that the following theorem is true.

Given a conic Σ and a parabola Σ_2 touching the axes of Σ, the points of contact on Σ of the four common tangents to Σ, Σ_2 are such that the normals at those points are concurrent at a point on the directrix of Σ_2.

8. In Fig. 1 the circle EFG goes through S_2, see 6 (1); let it cut CS_2, which is not shown in Fig. 1, in L.

Now the circle EFG cuts the director-circle of Σ orthogonally, $\because \Sigma$ is one of the conics through PP_1QQ_1.

$\therefore CL . CS_2 = CA^2 + CB^2$ (square of radius of director-circle), and S_2 is a fixed point; $\therefore L$ is a fixed point.

Thus the circle EFG is one of a coaxal system.

COROLLARY. From Th. 6. $CS_2 . CO = CS^2 = CA^2 - CB^2$,
and $CS_2 . CL = CA^2 + CB^2$.

Thus $CO/CL = (CA^2 - CB^2)/(CA^2 + CB^2)$.

Thus, if instead of Σ we take any conic similar to Σ and with its axes along CA, CB, the point L will be fixed.

Hence if the circle, centre O, is fixed, then for these conics similar to Σ the circles such as EFG form a coaxal system, for they cut OL at L and also at another fixed point L_1, since they cut the director-circle of the circle-centre O orthogonally.

***Note.*—The Apollonian hyperbola occurs also in the following theorem. PQ is any chord of a conic through a fixed point O; the circle on PQ as diameter cuts the conic again in P_1Q_1. Then the locus of the meet of PQ, P_1Q_1 is the Apollonian hyperbola of O.**

If instead of a central conic originally we take a parabola, Theorem 1 is more easily proved by Euclidean methods. The proof is quite simple. The other theorems, of course, require modification. W. F. BEARD.

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829. July 4, 1833. Newton was a great man, but you must excuse me if I think it would take many Newtons to make one Milton.—S. T. Coleridge, *Table Talk and Omniana*.

MATHEMATICAL NOTES.

1010. [L^1. 1. a.] *The Definition of a Conic Section.*

It is possible to give a slightly more general definition of a Conic Section than the one known as the focus-directrix definition.

DEFINITION. A Conic Section is the locus of P (Fig. 1) when MP has a constant direction, and P is related to two fixed points Z and S in such a way that

$$e^2 . ZM^2 - SM^2 = MP^2. \quad \text{.........................(1)}$$

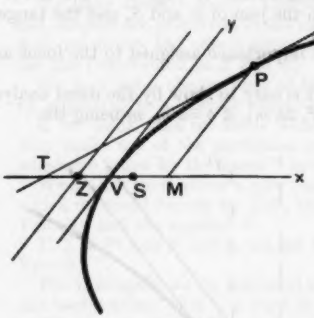

FIG. 1.

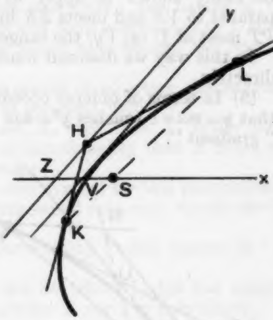

FIG. 2.

We show the use of this definition for the case of a parabola by taking $e=1$ so that we have

$$ZM^2 - SM^2 = MP^2. \quad \text{..................................(2)}$$

Clearly $MP=0$ when $ZM=SM$ ($=a$, say). This gives the vertex, V, of the parabola. It is to be noted that ZM^2 means only the product $ZM . ZM$, but that if $P\hat{M}S$ is 90°, this product may be interpreted in terms of the "square on ZM"; and if we remember Pythagoras' theorem, the equality (2) gives the focus-directrix definition of the parabola.

Without restricting PMS to be a right angle, take Vx, Vy as axes and let the coordinates of P be (x, y).

From (2) we get the equation of the parabola as:

$$(a+x)^2 - (a-x)^2 = y^2,$$

or

$$4ax = y^2. \quad \text{......................................(3)}$$

From (3) the following properties of the graph follow at once:

(1) Since $y = \pm\sqrt{4ax}$, the graph is symmetrical about Vx, or Vx is a diameter; the factor y^2 tells us that Vy is a tangent.

(2) If $y_1^2 = 4ax_1$, then $\dfrac{dy_1}{dx_1} = \dfrac{2a}{y_1}$, and the equation of the tangent at (x_1, y_1) is $y - y_1 = \dfrac{2a}{y_1}(x - x_1)$ or $yy_1 = 2a(x + x_1)$.

(3) If in $yy_1 = 2a(x+x_1)$ we put $y=0$ we get $x = -x_1$, or $VT = VM$ (Fig. 1).

(4) In the usual way we deduce that, if $y_1^2 = 4ax_1$, then $yy_1 = 2a(x+x_1)$ is the chord of contact or polar of (x_1, y_1). Hence if $(-a, y_1)$ is any point H on ZH, the chord of contact of H is $yy_1 = 2a(x-a)$, and if $y=0$ this gives $x=a$ (Fig. 2).

Hence the chord of contact of any point on $x=-a$ passes through S, and hence ZH is the polar of S.

Although this property is usually associated only with the directrix and focus, we see that it holds when ZH and S are not what are commonly known as directrix and focus.

The polar-pole relationship is further stressed if we observe that, since $VT=VM$ (Fig. 1), the points Z, V, S and the point at infinity on VS form a harmonic range; ZVS is a diameter when the fourth point of the harmonic range is at infinity.

Other properties usually associated with the focus-directrix definition may be easily shown to apply to the configuration of Fig. 3. Thus if PN is parallel to VS and meets ZN in N, then the join of N and S, and the tangent PT meet at U on Vy, the tangent at V.

In this way we discount much of the importance assigned to the focus and directrix.

(5) In terms of oblique coordinates, it is easy to show by the usual analysis that $y=mx+c$ touches $y^2=4ax$ at $(a/m^2, 2a/m)$, if $c=a/m$, m being the "gradient".

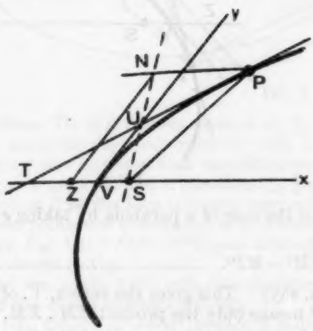

FIG. 3.

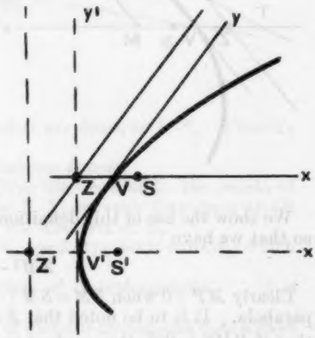

FIG. 4.

Quite apart from any advantage that may be possessed by the definition of a Conic Section made at the outset, the simple analysis that follows shows clearly both the strength and the weakness of the procedure often adopted of defining a "point" as a number pair (x, y); and a parabola as the class of "points" that satisfy the equation $y^2=4ax$. The strength lies in the fact that important known properties of a parabola may be easily written down or reproduced by simple analysis. The weakness lies in the fact that we cannot completely identify the above "parabola" with what we know as a geometric parabola until we show that, relative to Vx, Vy (Fig. 4) there are one pair of axes $V'x'$, $V'y'$ that are at right angles, and that the equation relative to these axes also has the same form as $y^2=4ax$.

We express the same idea by saying that the abstract definition of "parabola" indicated above makes no mention of a right angle.

Algebra and Geometry are distinct, and if we use one to assist the other we may mistake the object of both if we identify the terms of each.

B. NAYLOR.

1011. [V. 1. a.] *Unit Force.*

It is time, as I think Professor Sommerville implies in his letter,* that such words as "Poundal", "Poundem", "Poundweight", etc., were superannuated.

* *Math. Gazette*, Dec. 1930.

The name "Poundal" for "Unit Force" is unsuitable for the reason that it gives precedence to "Pound" to the exclusion of "Ft" and "Sec". If it is necessary to have a word-expression for "Unit Force", "Ft" and "Sec" should enter on a footing of equality with "Pound". We secure this, and at the same time perpetuate the name of the inventor of "Unit Force" if we use the term "Newton" as a synonym for $\frac{\text{"Lb. Ft"}}{\text{Sec}^2}$.

Further, we could appropriately call the Earth-Force on a Mass of 1 Pound by the name "Einstein", so that we have

$$1 \text{ Einstein} = g \,.\, \text{Newton.}^*$$

We have already many examples of this procedure; the names of Volta and Ampere are preserved in the names Volt and Amp (contracted to V and A). And, no doubt, the names above suggested would soon become the Newt and the Ein (contracted to N and E).

We may get a clearer view of the consequences of adopting this suggestion by appealing to a simpler example—that of constant Velocity.

If we think that we know what Distance, X, and Time, T, really are, we may make use of the particular relation or functionality that X bears to T, which we know by the name "proportionality".

We do not need either a new name or a new symbol for this proportionality which we could denote by X/T, but, for purposes of economy, we use the word Velocity and the symbol V.

If X is Ft and T is Sec, we get the proportionality Ft/Sec, known as "Unit Velocity".

For this again we do not need a name, but Sir Oliver Lodge has suggested the useful word "Vel", a word in which neither X nor T is mentioned.

Whether or not a name is justified on economic or scientific grounds, such names as this do meet a mental need of the student—they help the crystallisation of the concept in his mind.

But whatever views we may have on the advisability of introducing new names or of rejecting old ones it remains true that we must come to a decision with an eye on the needs of scientific intercourse rather than on the needs of everyday intercourse. V. NAYLOR.

1012. [K[1]. 11. e.] *Oriented Circles.*

The theorem of Hart, considered by Prof. Neville in his interesting note in the July (1930) number of the *Gazette*, can be expressed in another way.

§ 1. Let us give our circles a sense of rotation, and our lines a sense of direction; so that each line, in the ordinary sense, is replaced by two *oriented lines*, one in one direction, the other in the opposite direction, and similarly for circles.

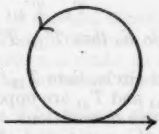

FIG. 1.

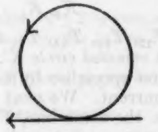

FIG. 2.

If an oriented line touches an oriented circle and the senses of the line and circle are related as in Fig. 1, the oriented line is called a *proper* tangent to the circle; but if these senses are related as in Fig. 2, it is called an "improper" tangent. The word "tangent", used without qualification, shall

* It is convenient to use capital letters for magnitudes and small letters for measures, *e.g.* $G = g$ ft./sec.^{2}

always mean "proper" tangent. If we say an oriented line "touches" an oriented circle, we shall mean it is a *proper* tangent.

When an oriented circle shrinks to a point, the orientation may be considered to disappear, and every oriented line through the point is both a proper and an improper tangent. *A circle which is properly tangent to an oriented line and also to its opposite oriented line must be a point-circle.*

Two oriented circles have two (proper) common tangents, and two improper common tangents, if they are mutually external and not point-circles.

§ 2. We can now state Hart's theorem as follows (see Coolidge, *Circle and Sphere*, p. 364):

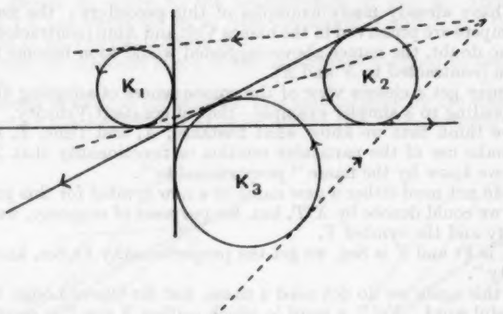

FIG. 3.

If K_1, K_2, K_3 be oriented circles and a common tangent of K_1 and K_2, a common tangent of K_2 and K_3, and an improper common tangent of K_3 and K_1 meet in a point, then the other common tangent of K_1 and K_2, the other common tangent of K_2 and K_3, and the other improper common tangent of K_3 and K_1 also meet in a point, or are parallel.

Following Müller (Einige Gruppen von Sätzen, *Jahresber. der Deuts. Math. Ver.* Bd. 20, 1911), we shall show that this is a special case of the general theorem which follows. In the next section we shall consider the general theorem.

If K_1, K_2, K_3, K_4 be oriented circles and

K_1, K_2 *have common oriented tangents* T_{12}, T'_{12},
K_2, K_3 .... T_{23}, T'_{23},
K_3, K_4 .... T_{34}, T'_{34},
K_4, K_1 .... T_{41}, T'_{41},

and if T_{12}, T_{23}, T_{34}, T_{41} touch an oriented circle K, then T'_{12}, T'_{23}, T'_{34}, T'_{41} touch an oriented circle K'.

We first specialise by taking K to be a point-circle, then T_{12}, T_{23}, T_{34}, T_{41} are concurrent. We next suppose also that T_{34} and T_{41} are opposite oriented lines, *i.e.* the same line, T say, taken with its two orientations. Then by § 1, K_4 becomes a point-circle on this line T. Call this point-circle K_0.

We then have K_1, K_2, K_3 oriented circles and

K_1, K_2 have common oriented tangents T_{12}, T'_{12},
K_2, K_3 .... T_{23}, T'_{23},

K_3, K_1 have an *improper* common tangent T. K_0 is a point on T.

The theorem states that: *if T_{12}, T_{23}, T are concurrent, then T'_{12}, T'_{23} and the other tangents T'_{34}, T'_{41} from K_0 to K_1 and K_3 touch an oriented circle.*

As a final specialisation we take the point K_0, which already lies on one common improper tangent of K_3, K_1, to lie on the other also. Then T'_{34}, T'_{41} become two oppositely oriented lines along that other common improper tangent. Hence, by § 1, the last oriented circle mentioned in the previous theorem becomes a point, and hence T'_{12}, T'_{23} and the second common improper tangent of K_3, K_1 concur. And this gives Hart's theorem.

§ 3. We now consider the general theorem of § 2 by the method, in itself interesting, of *cyclographic depiction* associated with the names of Fiedler and Müller.

The oriented circles in a given fixed plane a can be put into one-to-one correspondence with the points of space. For convenience, take a horizontal, and consider a circle of radius r and centre C on a. We make this circle correspond to a point of space at a distance r from C, and vertically above or below C according to the orientation of the circle. Thus the point which represents the oriented circle is the vertex of a cone through the circle, with a semi-vertical angle of 45°. A tangent plane to this cone cuts the plane a in a tangent to the circle. A point-circle, centre C, on a is represented by the point C itself.

Since the cones have their axes parallel, and their semi-vertical angles equal, they all cut the plane at infinity in the same conic, Γ, say. The plane through a common tangent to two oriented circles, and through their representative points, is a tangent plane to this conic Γ. Thus the common tangents to two oriented circles are the cuts of a with the two tangent planes to Γ that pass through the line which joins the points representing the oriented circles.

Hence the general theorem of § 2 is equivalent to:

If P_1, P_2, P_3, P_4 be four points in space, and t_{12}, t'_{12} be the tangent planes, to a fixed conic Γ, which pass through the line P_1P_2, and t_{23}, t'_{23}, t_{34}, t'_{34}, t_{41}, t'_{41}, have similar meanings, and if t_{12}, t_{23}, t_{34}, t_{41} meet in a point, then t'_{12}, t'_{23}, t'_{34}, t'_{41} also meet in a point.

Γ here clearly is an envelope of planes and is thus a degenerate quadric envelope. We next assert that the above theorem is true when Γ is *any* quadric envelope of planes.

To prove this, we will take the dual of the last theorem, Γ being now a general quadric envelope. This dual is

If $P_1P_2P_3P_4$ be a skew quadrilateral and P_{12}, P'_{12} be the cuts of the line P_1P_2 with a given quadric locus, and P_{23}, P'_{23}, P_{34}, P'_{34}, P_{41}, P'_{41} have similar meanings, and if P_{12}, P_{23}, P_{34}, P_{41} lie on a plane, then P'_{12}, P'_{23}, P'_{34}, P'_{41} also lie on a plane.

This is easily shown. For the points P_{ij}, P'_{ij}, eight in number, all lie on the quadric, and they also lie on the quadric made up of the plane pair $P_1P_2P_3$, $P_4P_2P_3$, and on the quadric made up of the plane pair $P_2P_1P_4$, $P_3P_1P_4$. They are thus eight associated points; hence if four are coplanar, so are the other four.

By means of the last theorem, the general theorem of § 2 is connected up with a well-known theorem on the eight cuts of four circles taken in order: if four cuts are on a circle, so are the other four.

This note by no means exhausts the contents of Müller's paper. H. G. F.

1013. [R. 4. a.] *Note on the Stability of Rolling Displacements.*

In the proof of the formula $\frac{1}{h} \gtrless \frac{1}{\rho} + \frac{1}{\rho'}$ as the condition for stability or instability of rolling displacements, it is usual to represent two near normals as intersecting at the centre of curvature and as of equal lengths ρ, in fact to assume that the curves in question may be replaced by their circles of curvature. This neglect of infinitesimal differences is likely to be disconcerting to the student, coming as it does after the discussion of Virtual Work and Stability, in which the importance of infinitesimals of the second order has been made

manifest. It seems preferable to avoid all question of orders of magnitude, as follows.

In *any* displaced position let the coords. of the point of contact P be (xy), $(x'y')$ referred to axes attached to the moving curve and fixed curve respectively. With the usual meanings of s and ψ, s is the same for both curves and the common normal at P makes angles ψ, ψ' with the axes OY, OY'.

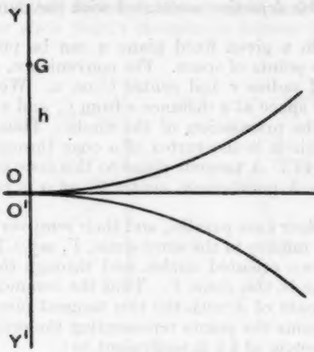

FIG. 1.

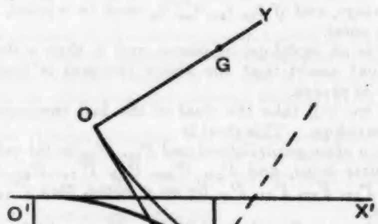

FIG. 2.

If z is the height of G above O',

$$z = -y' - y\cos(\psi+\psi') + x\sin(\psi+\psi') + h\cos(\psi+\psi').$$

All the variables being functions of s, we have, writing

$$\frac{dx}{ds} = \cos\psi, \quad \frac{d\psi}{ds} = \frac{1}{\rho}, \text{ etc.}$$

$$\frac{dz}{ds} = -\sin\psi' - \sin\psi\cos(\psi+\psi') + y\sin(\psi+\psi')\cdot\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$$
$$+\cos\psi\sin(\psi+\psi') + x\cos(\psi+\psi')\cdot\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$$
$$-h\sin(\psi+\psi')\cdot\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$$
$$=\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)\{x\cos(\psi+\psi')+y\sin(\psi+\psi')-h\sin(\psi+\psi')\}$$
$$=\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)(F), \text{ where } \rho \text{ and } \rho' \text{ relate to } P.$$

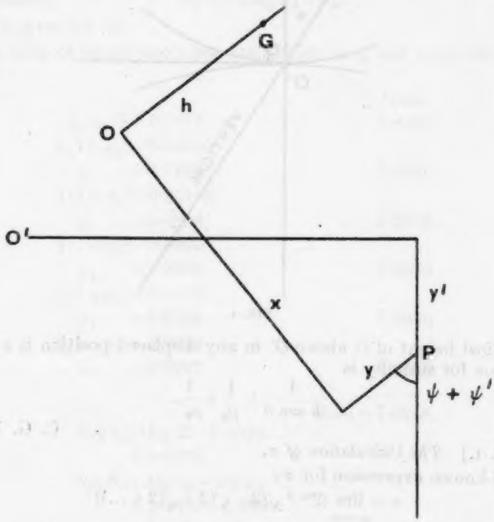

FIG. 3.

In the equilibrium position, $x=y=\psi=\psi'=0$, giving $F=\frac{dz}{ds}=0$.

$$\frac{d^2z}{ds^2}=(F)\cdot\frac{d}{ds}\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$$
$$+\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)\left\{\cos\psi\cos(\psi+\psi')-x\sin(\psi+\psi')\cdot\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)+\sin\psi\sin(\psi+\psi')\right.$$
$$\left.+y\cos(\psi+\psi')\cdot\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)-h\cos(\psi+\psi')\cdot\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)\right\}.$$

In the equilibrium position, $F=0$, etc., giving

$$\frac{d^2z}{ds^2}=\left(\frac{1}{\rho_0}+\frac{1}{\rho_0'}\right)\left\{1-h\left(\frac{1}{\rho_0}+\frac{1}{\rho_0'}\right)\right\},$$

so that, for stability,

$$\frac{1}{h}>\frac{1}{\rho_0}+\frac{1}{\rho_0'}.$$

If the position of equilibrium is one in which the common normal at O' is inclined to the vertical, replace h by $h \sec a$.

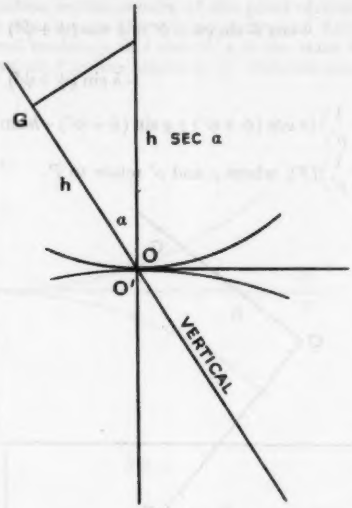

FIG. 4.

The vertical height of G above O' in any displaced position is $z \cos a$, and the condition for stability is

$$\frac{1}{h \sec a} > \frac{1}{\rho_0} + \frac{1}{\rho_0'}.$$

C. G. PARADINE.

1014. [X. 1.] *The Calculation of* π.

The well-known expression for π:

$$\pi = \lim_{n \to \infty} \{2^{n-1} \sqrt{(2 - \sqrt{(2 + \sqrt{(2 + \ldots)}}}$$

is inconvenient for calculation because the surd expression is very small and in finding it we lose many significant figures. This can be avoided by rationalisation of the numerator. We can, however, arrive at the same result in the following way, which is, I think, more instructive.

Let c_n be the length of a chord of a regular polygon of 2^n sides inscribed in a circle of radius 1. Then it is easy to show that

$$4 - c_n^2 = 2 + \sqrt{(4 - c^2_{n-1})}.$$

Let $\quad 4 - c_n^2 = 4y_n^2,$

then $\quad y_n^2 = \tfrac{1}{2}(1 + y_{n-1}),$

and hence

$$\begin{aligned} c_n^2 &= 4 - 4y_n^2 \\ &= 2 - 2y_{n-1} \\ &= \frac{4 - 4y_{n-1}^2}{2 + 2y_{n-1}} \\ &= c_{n-1}^2 / 4y_n^2 \\ &= \frac{1}{2^{2n-4} y_n^2 y_{n-1}^2 \cdots y_2^2}. \end{aligned}$$

Therefore the perimeter of the inscribed polygon is

$$s_n = 2^{n-1}c_n = \frac{2}{y_n y_{n-1} \cdots y_2}. \qquad \text{(i)}$$

Further, if t_n is the length of a side of a regular polygon of 2^n sides circumscribed about a circle of unit radius, it is easily shown that

$$t_n = \frac{2c_n}{\sqrt{4-c_n^2}} = \frac{c_n}{y_n},$$

and therefore the perimeter of the circumscribed polygon is

$$S_n = 2^{n-1}t_n = 2^{n-1}c_n/y_n = s_n/y_n.$$

Consequently $$s_n < \pi < s_n/y_n = S_n,$$

where s_n is given by (i).

With a table of square roots the calculation of s_n and s_n/y_n can be set out as follows:

	Logs.
$y_2 = \sqrt{\tfrac{1}{2}} = 0{\cdot}7071$	$\bar{1}{\cdot}8495$
$\tfrac{1}{2}(1+y_2) = 0{\cdot}85355$	
$y_3 = 0{\cdot}9239$	$\bar{1}{\cdot}9656$
$\tfrac{1}{2}(1+y_3) = 0{\cdot}96195$	
$y_4 = 0{\cdot}9808$	$\bar{1}{\cdot}9916$
$\tfrac{1}{2}(1+y_4) = 0{\cdot}9904$	
$y_5 = 0{\cdot}9952$	$\bar{1}{\cdot}9979$
$\tfrac{1}{2}(1+y_5) = 0{\cdot}9976$	
$y_6 = 0{\cdot}9988$	$\bar{1}{\cdot}9995$
$\tfrac{1}{2}(1+y_6) = 0{\cdot}9994$	
$y_7 = 0{\cdot}9997$	$\bar{1}{\cdot}9999$
	Sum $= \bar{1}{\cdot}8040$,

whence $$\log s_n = \log 2 - \bar{1}{\cdot}8040 = 0{\cdot}4970,$$

and $$\log S_n = \log s_n - \log y_n = 0{\cdot}4970 - \bar{1}{\cdot}9999 = 0{\cdot}4971,$$

which give $s_n = 3{\cdot}141$ and $S_n = 3{\cdot}142$.

In the above work I have purposely avoided trigonometry because I believe it is very useful to be able to show a student how π can be approximated to using nothing more than Pythagoras' theorem and elementary algebra.

H. V. Lowry.

The College of Technology, Manchester.

1015. [L¹. 9. d.] *Circumference of Ellipse.*

Encyclopaedias give as an approximate circumference of an ellipse whose semi-axes are a and b

$$\pi\left\{\frac{a+b}{2} + \sqrt{\frac{a^2+b^2}{2}}\right\}. \qquad (A)$$

I prefer as simpler, and more accurate except for ellipses so long that both approximations fail,

$$\pi\{1\tfrac{1}{2}(a+b) - \sqrt{ab}\}. \qquad (B)$$

It would interest me to hear whether this approximation is already known and used.*

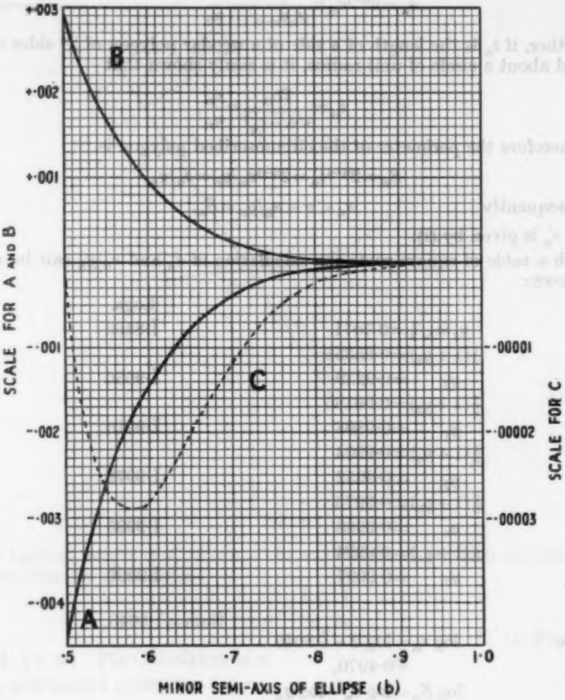

Fig. 1.

If A and B are mixed in the ratio 2 : 3, a rather less simple but remarkably close approximation results :

$$\pi\left\{1{\cdot}1(a+b)+{\cdot}4\sqrt{\frac{a^2+b^2}{2}}-{\cdot}6\sqrt{ab}\right\}. \quad\ldots\ldots\ldots\ldots\ldots\ldots(C)$$

To appreciate the accuracy of this, notice that the graph of its error is drawn on *one hundred* times the vertical scale of the other error-graphs.

Can anyone tell me of a good approximation to the circumference of a long ellipse ?

W. Hope-Jones.

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830. April 20, 1811. He wished some portion of mathematics was more essential to a degree at Oxford, as he thought a gentleman's education incomplete without it, and had himself found the necessity of getting up a little, when he could ill spare the time. He every day more and more lamented his neglect of them when at Cambridge.—S. T. Coleridge, *Table Talk and Omniana.*

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* [The approximation (B) is given by Rothe, *Höhere Mathematik*, Teil II, p. 121, § 7, where it is shewn that the error is of the order of $10^{-3} \cdot e^8$ where e is the eccentricity. It is given without proof in some English books on Practical Mathematics. Ed.]

REVIEWS.

The Principles of Quantum Mechanics. By P. A. M. DIRAC. 17s. 6d. Pp. x + 257. 1930. (Oxford: At the Clarendon Press.)

An eminent European physicist, who is fortunate enough to possess a bound set of reprints of Dr. Dirac's original papers, has been heard to refer to them affectionately as his "bible". Those not so fortunate have now at any rate an opportunity of acquiring a copy of the authorised version. In this book we have a complete account of the new concepts of theoretical physics, a formulation of Nature's fundamental laws as far as this has been possible, and a description of the mathematical machinery which has been set up to deal with the observable effects of these laws. The successes of the present formulation are so great that it is unlikely that any serious change will have to be made in the underlying assumptions on which the theory rests, at any rate for the purpose of explaining the ordinary physical and chemical properties of matter.

A new system of dynamical equations is given, which are generalisations of the Newtonian or Hamiltonian equations; in fact, the old are seen to be special cases of the new, corresponding to the case when Planck's constant h is zero. The new equations of motion have, however, to be supplemented by other equations, the so-called quantum conditions, before a dynamical system is mathematically completely specified.

The book contains Dr. Dirac's philosophy of the relation of theoretical and experimental physics. He believes that the main object of theory is to determine the possible results of an experiment, and to determine the probability that any one of these results will actually occur under given conditions. He regards it as quite unnecessary that any satisfying description of the whole course of the phenomena should be given. A mathematical machine is set up, and without asserting or believing that it is the same as Nature's machine, we put in data at one end and take out the results at the other. As long as these results tally with those of Nature, (with the same data or initial conditions) we regard the machine as a satisfying theory. But so soon as a result is discovered not reproduced by the machine, we proceed to modify the machine until it produces the new result as well.

One might have hoped that the object of the theoretical physics was rather more ambitious than Dirac is willing to allow, and that the steady march forward of physics was taking us further and further forward to a knowledge of the nature of things. But the theoretical physicist, it would seem, must for ever abandon any hope of providing a satisfying description of the whole course of phenomena. He must concentrate on the facts of experience, the islands, as it were, which project up above the sea of the unknown into the light of experience, but the nature of the underlying substratum, the sea-bed, of which the islands are but the small parts, must for all time remain beyond his ken.

The machinery, described in Dr. Dirac's book, is a complicated symbolic calculus and the fodder for it consists of "observables" and "states" (precisely defined at the beginning and precisely re-interpreted at the end). The "state" of an atomic or any other system refers to its condition throughout an indefinite period of time and is denoted by an abstract symbol ψ. An "observable", on the other hand, refers to a dynamic variable at a particular time. It also is denoted by an abstract symbol α. Nowhere is the exact nature of the symbols specified, nor is such specification necessary. All that is required is a knowledge of the axioms and rules which they satisfy and a knowledge of the connexion between equations involving them and physical conditions.

The main connecting link between the symbolic algebra and physical facts is that the average value of an observable α for the state ψ, when measured a large number of times (the system being reprepared each time in order that it may be in the proper state), is equal to $\phi\alpha\psi$, where ϕ is the conjugate imaginary of ψ.

The ψ's have a certain analogy with vectors, and so it is convenient to consider a space of as many dimensions as there are independent states of the system. To each axis corresponds an independent state, and any other state

may be considered as completely specified by the direction of a vector relative to these axes. An observable is like a linear operator, and, when operating on any independent state, it has the effect of rotating the corresponding axis relative to the original set of axes. An observable is represented by the directions of all these vectors, one for each axis, relative to the original set of axes. It is thus represented by a matrix. Matrices, like observables, satisfy all the laws of ordinary algebra except the commutative law of multiplication.

The same observables may be represented by matrices relative to another independent set of axes, that is, to another set of independent states. Transformations of this kind are called canonical transformations, and are of the greatest importance in quantum mechanics. Just as transformation theory was at the basis of the theory of relativity, so transformation theory (of a more general kind) is the essence of the new method in quantum mechanics. The important things of the world appear as invariants of transformations.

The first half of the book is given up to the general considerations just outlined, and the second half is concerned with applications to a number of physical problems of importance. There is an account of the new theory of the electronic structure of atoms, of collision problems and of radiation theory. The book concludes with an account of the relativity of theory of the electron, its achievements and limitations, and brings the reader in contact with some of the fundamental problems still unsolved.

It would be idle to pretend that the book is easy to read. While, with one or two isolated exceptions, every sentence is simply and lucidly and concisely expressed, the subject is so abstract, and the ideas so new, that the book requires close attention. But just as it repaid the effort to study books on relativity, written by the experts, so now a study of this book, which outlines advances in thought quite as important and as far-reaching as the ideas of relativity, will well repay the effort it costs. The highest praise that can be bestowed on the book is to say that it should be read by everyone who desires to keep in touch with modern physics.

This book is the first of an International Series of Monographs on Physics to be produced by the Oxford University Press, and is one of the most beautiful examples of the art of printing which the reviewer has seen.

J. E. LENNARD-JONES.

The Theory of Approximation. By DUNHAM JACKSON. (American Mathematical Colloquium Publications, Volume XI.) Pp. viii + 178. Price not stated. 1930. (American Mathematical Society, New York.)

In 1925, Professor Jackson lectured at the American Mathematical Society's Colloquium on certain aspects of the theory of approximation, *i.e.* on the theory of approximating to a function of a real variable by means of a finite sum of functions of given form. This theory covers a vast field, and, in his lectures, Professor Jackson chose certain topics in a few corners of this field in which he is personally interested and where he has made valuable contributions.

The theory had its starting point in Weierstrass's famous theorem concerning the uniform approximation to a function, continuous in an interval, by means of polynomials. This, too, is Professor Jackson's starting point, and his first chapter deals with approximation by polynomials and trigonometric polynomials and the allied topics of Fourier and Legendre Series. In the second chapter, the same work is carried out under less stringent conditions than that of continuity. The technique involved is always quite elementary, and, if some generality is lost, the presentation is always clear.

The third chapter deals with the Principle of Least Squares, in other words, with certain aspects of Parseval's Theorem. Whilst rather more knowledge is expected of the reader here, the analysis is again of a clear and elementary character. The fourth chapter deals with the interesting analogies between Fourier Series and the series of trigonometrical interpolation. The last chapter is rather disappointing; for in it the author has wandered very far from his theme, and shows how the notion of Functional Space is of value in Mathematical Statistics!

Unfortunately, the value of the book, with its clarity of exposition, is completely spoiled by an entire lack of references to original papers. Whilst one does not expect an extensive bibliography, it should not be necessary to have to turn to Pólya and Szegö to find out when and where Bernstein proved his theorem on the derivative of a trigonometric polynomial. Footnotes are quite cheap! E. T. C.

Examples in Elementary Algebra. By W. M. Deans. Pp. 36. 1s. 1930. (Blackie.)

A varied collection of exercises, up to and including the solution of the quadratic equation, in a handy form suitable for revision purposes.

Algebraic Charts. By Edgar Dehn. 3s. 6d. 1930. (Oxford Univ. Press.)

Six charts designed to solve quadratic and cubic equations with real or imaginary roots, and biquadratic equations with a real root. The solutions are given in general as the coordinates of the points of intersection of curves representing coefficients of the equation as functions of the roots. The charts themselves are marvels of ingenuity and draughtsmanship, but their accuracy depends almost entirely on the skill of the reader. They should be of practical use to the scientist and engineer, and of considerable theoretical interest to the mathematical student. E. L.

The Calculus. By H. H. Dalaker and H. E. Hartig. Pp. viii + 254. 11s. 3d. 1930. (McGraw-Hill.)

There seems to be no reason to suppose that this book will appeal to English teachers of the subject. It is excellently printed on good paper, and in the worked-out examples the authors have been careful to avoid mere substitution in formulae. But here its particular merits end. Like other American books on the Calculus, the space devoted to fundamentals is small; the concepts of functions, limits, continuity and differentiation are introduced and dismissed in fourteen not very closely printed pages. It is good to see the differential defined as the principal part of the infinitesimal increment of the function, but it is nowhere stated that this principal part must be linear in the increments of the independent variables, and for this reason the treatment of differentials of functions of more than one variable is a little obscure. Finally—a skeleton book, and the bones are very dry.

Number: The Language of Science. By Tobias Dantzig. Pp. viii + 260. 10s. 1930. (Allen and Unwin.)

It should be made clear at the outset that this book is in no sense a contribution to the mathematical analysis of the concept of number. To quote from the preface, it "deals with ideas, not with methods", and again, it presupposes no greater knowledge of mathematics "than that which is offered in the average high-school curriculum". The author states that the book is not intended to be a history of the subject, but the method of exposition adopted is, in the main, historical. We may perhaps most accurately describe it as a history, not of number, but of the evolution of the number-concept. The story starts with the number-sense possessed by men, some birds and—apparently—a certain type of wasp. It advances through the stage of finger-counting to the Hindu numeration, and thence by well-known steps to the age of Cantor, Dedekind and Weierstrass. This part of the book is well written and interesting, and would be very valuable to schoolboys specialising in mathematics, as well as to teachers who have allowed their knowledge of the history and philosophy of mathematics to rust. The author says what he has to say with a pleasant ease, and his facts are generally sound. It need only be added that, here and there, he permits himself a little gentle speculation, not infrequently too airy to be sound. For example, it is surely a little far-fetched to assert that if Fermat had only published his work, he would have completely anticipated the work of Descartes on analytical geometry and of Newton and Leibnitz on the Calculus.

That part of the volume to which reference has just been made constitutes eight chapters out of the twelve. The remaining four deal with Cantor's *Mengenlehre*, the antinomies and various points in the philosophy of number. These chapters are by no means as praiseworthy as the earlier ones. True, the subject matter is such that to write on it scientifically is not easy, and to write on it in a style suited to readers with a not very extensive acquaintance with the appropriate technique must be an even more difficult task. But Dr. Dantzig's chapters are more confused than they need have been, and some of his statements, if not inaccurate, are at least misleading. Few will be offended by the author's confession that he has not read all three volumes of *Principia Mathematica*, but Russell's *Introduction to Mathematical Philosophy* shews the lack of discrimination in Dr. Dantzig's remark that " on the side of the formalists are Hilbert, Russell and Zermelo ". A paper in the *Gazette* by the late F. P. Ramsey * gives a most lucid account of the various schools of thought and shows how the Russell-Whitehead ideas differ from the " formalism " of Hilbert and the " intuitionism " of Weyl and Brouwer. It is not claimed that a solution of the problems of mathematical logic has been reached along the line laid down in *Principia Mathematica*, and in an obituary notice of Ramsey, R. B. Braithwaite † points out that Ramsey himself " came to doubt its complete adequacy " ; nevertheless, the line of thought is a most important one, and its importance should not be concealed by confusing it with other, no doubt equally important, researches on the foundations of mathematics.

It has been thought necessary to criticize adversely this portion of Dr. Dantzig's work, as a corrective to the claim on the cover that " in this book is given an admirable exposition of the fundamental philosophical ideas which have revolutionised mathematical thought during the past twenty years ". This seems to us to disparage the best part of the book, that part wherein is presented an " admirable exposition " of the evolution of number up to the time of Cantor. For an exposition of the recent developments the reader must seek in other places, but he would do well to read this volume first, as would those who, without being themselves professional mathematicians in any sense, are yet interested in the philosophy and history of mathematics.

T. A. A. B.

School Certificate Trigonometry (with Mensuration). By J. J. Walton. Pp. vii + 195. With answers, 4s.; without, 3s. 6d. 1930. (Isaac Pitman.)

The author believes in " developing the trigonometrical sense through the geometrical." A wide range is covered in less than 200 clearly printed pages. Simple numerical trigonometry is dispensed with in the first two chapters, identities occur in chapters III and V, mensuration in IV and VIII, the solution of triangles in VII. Chapter X introduces the addition formulae and in the compass of some ten pages includes the $3A$ formulae and the value of sin 18°. The concluding four chapters treat of R, r, etc., sums and products, inverse functions and the general solution of a simple trigonometrical equation. There are ten pages of tables at the end.

The exercises are not numerous and the practical applications are certainly not overdone.

C. J. A. T.

GLEANINGS: AN APPEAL.

The Editor will be grateful for help in the filling up of odd corners. A precise reference should accompany every quotation.

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* *Math. Gazette*, xiii (1926), 185-194, " Mathematical Logic ".

† *Journal London Math. Soc.* 6 (1931), 77.

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD.
THE UNIVERSITY PRESS, GLASGOW.

(LONDON BRANCH.)

PROFESSOR LEVY delivered the Presidential Address on "The Dimensional Theory as a School Subject" on Saturday, December 7th, 1929, at Bedford College. There were 52 present, with Mr. Katz in the chair.

Professor Levy believed that by relating mathematical instruction to actuality the pupil could be convinced of the truth of what he was being taught. The subject of the address had been suggested by a question posed by his eight year old daughter who drank a quart of milk per day. "How many days," she enquired, "would the quart bottle last us if it was ten times as big, all ways ?" It was obvious to him, of course, at once, as it should have been to every one reasonably educated, that it would be nearly three years, but when the same enquiry was put by him to a well-known writer of no scientific training, he guessed about seven days. It seems lamentable that such an absurd answer should appear reasonable to a well educated man, and it argued something fundamentally lacking in our system of instruction. This particular want, the lecturer maintained, could be filled by bringing the pupil into contact at all ages and at all stages with the dimensional theory.

As an illustration of the relative variation of volumes and surfaces with linear dimensions, Professor Levy asked his hearers to imagine that they were out shopping and proposing to buy a hot-water bottle, one which would remain hot as long as possible. "Would you buy the largest obtainable or would you buy a small one ?" he asked, and he showed some amused surprise when a lady produced the right answer. Similar principles were applied to illustrate the sizes of ears and other protuberances on the bodies of animals living near the Poles and near the Equator.

Passing to other examples which involve the conceptions of stress, he dealt with the question whether when a model of a structure, such as a bridge, was strong enough on one scale it would be strong enough when enlarged to the full scale, or when an engine working properly as a model could be expected to do so when enlarged. As an illustration of the latter he took the case of an oscillating pendulum, whose image on a different scale was thrown on a cinematograph screen, and pointed out how the picture would not represent a real oscillating pendulum unless the film were run at a speed related in a simple way to the increase in the linear dimensions. Under the same heading of "stress" and the area of section that takes it, he gave a series of illustrations on the limiting statures of men, trees, etc. Simple consideration of dimensions showed that the maximum velocity obtainable by a falling parachute was very roughly independent of the size, whereas for a falling mass it increased linearly.

Turning to the fact that all physical equations are homogeneous, Professor Levy dealt with a number of dynamical and other laws of variation, showing that they could be deduced from purely dimensional considerations. He illustrated this by determining the acceleration towards the centre of a particle traversing the circumference of a circle ; the pitch of a note obtained by blowing through an open key used as a whistle ; the note of a plucked string ; the period of swing of a pendulum, and a number of other formulae on the speed of waves. The lecturer stressed the advantages of using equations in non-dimensional form, writing, for example,

$$s=ut+\tfrac{1}{2}at^2$$

in the form
$$\frac{s}{ut}=1+\frac{1}{2}\frac{at}{u},$$

so that the latter could be illustrated in all cases by a single linear graph of $\frac{s}{ut}$ against $\frac{at}{u}$.

F. C. BOON, Hon. Sec.

LONDON BRANCH.

THE Annual Meeting was held at Bedford College on Saturday, 1st February. There was an attendance of 48. Mr. Katz was in the chair.

The Branch was glad to welcome as a visitor to the meeting Miss M. O. Stephens, a Secretary of the Manchester Branch.

The Committee reported that the numbers stood at 142 Full, and 91 Associate Members, of whom 37 had been enrolled during the year. [It was added that 15 others had joined since 1st January, 1930.] The attendance at meetings was increasing, the average for 1929 being 56, as compared with 50 in 1928.

The Branch expressed its thanks to the authorities of Bedford College and Mercers' School for the use of rooms, and to Bedford College for arranging tea facilities after meetings.

The Treasurer's statement reported a satisfactory financial position.

The Officers for the year were then elected, and the meeting proceeded to the discussion of the following topics:

(1) "Do we over-emphasize Style?"—Introduced by Miss R. H. King.

(2) "Mnemonics for Trigonometrical Formulae."—Mr. Bickley.

(3) "The Two Meanings of $\int ydx$."—Mr. Daltry.

(4) "A direct proof that the greater angle in a triangle is subtended by the greater side."—Mr. Kearney.

In the lively discussion which followed on these topics, Messrs. Styler, Inman, Hope-Jones, Dr. White, Prof. Roberts, Prof. Lodge, Messrs. Barnard and Katz took part.

At Bedford College, Regent's Park, N.W. 1, at 3 p.m.

Mar. 22nd. Discussion "Are we satisfied with the present Syllabus in Mathematics for the General School Certificate?" Opener: C. L. BEAVEN.

SUMMER MEETING.—It is hoped to a visit a Gramophone Record Factory.

The Secretaries will be glad to receive:

1. Suggestions for discussion at the Annual Meeting.
2. Names of Members willing to take part in the Discussion on March 22nd.

BRISTOL BRANCH.

THE membership of the Branch during the session 1928-9 was 34, including 15 members and 19 associates.

The papers read were as follows:

Oct. 21st, 1928. "The Calculus and the Teaching of Applied Mathematics."—Professor H. R. HASSÉ.

Nov. 16th, 1928. "Mediaeval Methods of Multiplication and Division."—Mr. G. W. HINTON.

Feb. 1st, 1929. "Theories of Light and Theories of Matter."—Professor J. E. LENNARD-JONES.

March 1st, 1929. "Do Arithmetic and Geometry form a Single Special Ability?"—Mrs. OLDHAM.

G. W. HINTON, Hon. Sec.

THE YORKSHIRE BRANCH.

AT a meeting held on Saturday, 8th February, at the University, Leeds, Dr. Wilson (of Liverpool University) read an interesting paper on "Ramanujan," in which he dealt with his life and work, giving illustrations both from his published papers and also from his unpublished note-books. Miss Spalding, M.A., and Miss Baugh, M.A., of the Training College, Bingley, opened an interesting discussion on the course of Elementary Mathematics which should be taken by those pupils who are intending to take up teaching, with special reference to Arithmetic. The President, Mr. Oldfield, was elected to serve on the Council of the Parent Association as Branch Representative.

LIVERPOOL MATHEMATICAL SOCIETY.

At the Annual Meeting of the Liverpool Mathematical Society, held at the University of Liverpool, on Monday, 27th May, 1929, it was decided to secure affiliation with the Mathematical Association.

For the Session 1929-1930, the Officers were elected as follows:

President—Dr. S. F. Grace, The University, Liverpool.
Vice-President—Miss M. Ralph, The Queen Mary High School, Liverpool.
Hon. Treasurer—R. O. Street, The University, Liverpool.
Hon. Secretary—R. Baldwin, The Grammar School, Wallasey.
Council—H. A. Baxter, Miss Cobbam, Miss McCormick, Prof. J. Proudman, W. J. Walker, Dr. B. M. Wilson.

The Society has about thirty-five members, of whom thirteen are members of the Mathematical Association.

Two meetings of the Society have been held in the Autumn Term.

At the first meeting, held at the University on Monday, 28th Oct., 1929, the President, Dr. S. F. Grace, gave a paper entitled " The Elements of Projective Geometry by means of Ruler and Compass."

The second meeting was held on Monday, 25th Nov., in the Physics Theatre of the University, when Prof. J. Rice gave a lecture on " The Teaching of the Laws of Motion."

There was an attendance of about thirty at each meeting.

R. Baldwin, Hon. Sec.

MANCHESTER AND DISTRICT BRANCH.

President—Miss E. F. Edwards.

On 22nd October, 1929, the Annual Meeting was held for the election of Officers. An interesting lecture was given by Mr. J. M. Child on "The Theory of Approximations," applied (i) to the use of logarithmic tables and (ii) to the approach to irrationals. At the second Autumn Meeting, held on 2nd December, the Branch received a welcome visit from Mr. C. V. Durell. His address on " Loci " was very stimulating, and the apparatus which he brought attracted much attention.

This term the joint meeting with the University Mathematical Society took place on Wednesday, 19th February, when Professor E. H. Neville lectured on " Limits in Geometry." On Friday, 21st March, the Branch will be addressed by Miss M. A. Hooke, Headmistress of Bradford Girls' Grammar School.

It has been decided that in future the election of Officers shall take place at the end of each session. This will necessitate the holding of another meeting during the present session, when we shall hope to follow the business meeting by a discussion of members' questions. Members are asked to add to the success of that meeting by sending as soon as possible to the Secretary suggestions of topics which may be generally interesting. M. O. Stephens, Hon. Sec.

QUEENSLAND BRANCH—Seventh Annual Report.

The Annual Meeting was held at the University on Friday, March 30th, 1928. Professor Priestley chose for his Presidential Address the subject: " The Influence of Mathematics on Human Thought."

During the year three General Meetings were held. At the first of these Miss E. Raybould, B.A., read a paper entitled " A Glance at the Foundations of Mathematics." At the second, Mr. J. P. McCarthy, M.A., read a paper on " Orthogonal Projections in Elementary Mathematics." The final meeting was devoted to a discussion, which Mr. S. Stephenson, M.A., opened, on the requirements in Mathematics in the University Public Examinations under the educational system about to be introduced.

The number of financial members is twenty-five, the same as for last year. Of these, eleven are full members of the Mathematical Association. Copies of *The Mathematical Gazette* come to hand regularly and are circulated among Associate members.

The statement of receipts and expenditure reveals a balance in hand of £10 12s. 3d., an increase of nearly three pounds for the year. This is partly accounted for by the fact that some members paid subscriptions in advance.

March 22nd, 1929. J. P. McCarthy, *Hon. Sec.- Treas*

THE MATHEMATICAL ASSOCIATION.

(*An Association of Teachers and Students of Elementary Mathematics.*)

"I hold every man a debtor to his profession; from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves, by way of amends, to be a help and an ornament thereunto."—BACON (Preface, *Maxims of Law*).

THE MATHEMATICAL ASSOCIATION, which was founded in 1871, as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

Its purpose is to form a strong combination of all persons who are interested in promoting good methods of teaching mathematics. The Association has already been largely successful in this direction. It has become a recognised authority in its own department, and is continuing to exert an important influence on methods of examination.

The Annual Meeting of the Association is held in January. Other Meetings are held when desired. At these Meetings papers on elementary mathematics are read and discussed.

Branches of the Association have been formed in London, Bangor, Yorkshire, Bristol, Manchester, Cardiff, the Midlands (Birmingham), the North-Eastern District (Newcastle-upon-Tyne), New South Wales (Sydney), Queensland (Brisbane), and Victoria (Melbourne). Further information concerning these branches can be obtained from the Honorary Secretaries of the Association.

"The Mathematical Gazette" (published by Messrs. G. BELL & SONS, LTD.) is the organ of the Association. It is issued at least six times a year. The price per copy (to non-members) is usually 2s. 6d. each. The *Gazette* contains—

(1) ARTICLES, mainly on subjects within the scope of elementary mathematics;

(2) NOTES, generally with reference to shorter and more elegant methods than those in current text-books;

(3) REVIEWS, written when possible by men of eminence in the subject of which they treat. They deal with the more important English and Foreign publications, and their aim is to dwell on the general development of the subject, as well as upon the part played therein by the book under notice;

(4) QUERIES AND ANSWERS, on mathematical topics of a general character.

THE MATHEMATICAL ASSOCIATION.

NORTH-EASTERN BRANCH.

THE Annual General Meeting was held at Newcastle on Saturday, 8th February, 1930, when Professor P. J. Heawood of Durham University was elected President for the ensuing two years. The Vice-Presidents were re-elected, with the addition of Mr. J. Strachan the retiring President. Miss Dow, who was re-elected Hon. Treasurer, presented the accounts, which showed a balance in hand of £15 18s. 9d. The Secretaries' report for the year stated that there were 64 members and 14 associates. Meetings had been held during the year as under: on 16th March, at Armstrong College, when a paper was read by Professor Curtis on "Modern Scientific Research"; 4th May, in the University Lecture Room, Durham, when papers were read by Professor Heawood on "A Probability Problem involving the use of the Binomial Series,"* and by Mr. Van der Heyden entitled "Fads and Fancies in Mathematical Teaching"; on 19th October, at Dame Allan's School, Newcastle, when papers were read by Professor Havelock on "Charles Hutton and his Times," and by Mr. A. K. Wilson on "Accuracy in Elementary Mathematics"; on 7th December, at Armstrong College, when Professor Whittaker of Edinburgh University visited the Branch and gave a paper on "The Doctrine of Parallels in recent theories of Geometry and Physics."

At the Annual Meeting a paper was read by Mrs. Wardley-Smith entitled "Psychology and Mathematics."

Miss M. Waite was re-elected as one of the Hon. Secretaries, and Mr. J. W. Brooks was elected in place of Mr. Wilson, who resigned.

A. K. WILSON, Hon. Sec.

LONDON BRANCH.

ON Saturday, 1st March, at Bedford College, Prof. Roberts addressed a meeting of 57 members, with Mr. Katz in the Chair, on "Some Difficulties in Teaching Mechanics." He found that pupils were not so interested in Statics as in Dynamics, and that in Statics bad habits were easily acquired. In solving problems the Laws of Statics were not the only things to be observed; there were Local Rules, the most important of which was that a separate—really separate—diagram should be drawn for each body under consideration. In Dynamics energy should be introduced early. The energy equation is more important than the f equations, it often obviates the necessity of integration, and f is constant only in a few special cases. He also dealt with some difficulties in circular motion and impact, and suggested ways of dealing with them.

Mr. C. V. Durell spoke with reference to the first year's course. He advocated the use of apparatus which helped pupils to visualise the real nature of the problem and believed that with its use Statics formed the easier introduction to the subject. Pupils had generally dealt with a variable f in the use of graphs in the Algebra course. By regarding weight as the typical force and dealing with forces in one dimension considerable ground could be covered and the chief principles grasped in the first year.

Prof. Jolliffe, Prof. Lodge, Messrs. Bickley, Atkinson, Kearney, Inman, Hills, and Miss Lester also joined in the discussion. F. C. BOON, Hon. Sec.

* *Gazette*, xiv., p. 567.

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Bauer, G. N. *Mathematics Preparatory to Statistics and Finance.* Pp. 337. 8s. 6d. net. 1929. (Macmillan Co.)

Beard, W. S. *Exercises in Algebra for Beginners.* Pp. 64. 10d. Answers, 6d. net. 1929. (Bell.)

Bieberbach, L. *Lehrbuch der Funktiontheorie.* Band I. 3rd edit. Pp. vii + 322. RM. 17. 1930. (Teubner.)

Bieberbach, L. *Analytische Geometrie.* Pp. iv + 120. RM. 6.60. 1930. (Teubner.)

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Borchardt, W. G. *Senior Algebra Test Papers.* With Answers. Pp. 87 + xv. 1s. 3d. 1930. (Rivingtons.)

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Chace, A. B. ; Manning, H. P. ; Archibald, R. C. ; Glanville, S. R. K. *The Rhind Mathematical Papyrus, British Museum* 10057 *and* 10058. *Photographic Facsimile, Hieroglyphic Transcription, Transliteration, Literal Translation, Free Translation, Mathematical Commentary, and Bibliography, B.M. Leather Roll.* In Two Volumes. n.p. 1927, 1929. (Math. Assoc. of America, Oberlin, Ohio.)

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Dalaker, H. H., and Hartig, H. *The Calculus.* Pp. viii + 251. 11s. 3d. net. 1930. (McGraw-Hill.)

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Duschek, A., and Mayer, W. *Lehrbuch der Differentialgeometrie.* Vol. I. (Duschek). *Kurven und Flächen im Euklidischen Raum.* Pp. viii + 250. RM. 17. Vol. II. (Mayer). *Riemannsche Geometrie.* Pp. viii + 245. RM. 17. 1930. (Teubner.)

Fry, T. C. *Elementary Differential Equations.* Pp. x + 255. 10s. 6d. net. 1929. (Macmillan.)

Gérard, L. *Sur le Problème de Malfatti, Le Pendule de Foucault, et autres questions d'Analyse et de Physique.* Pp. 64. n.p. 1929. (Vuibert.)

Gibbs, R. W. M. *The Adjustment of Errors in Practical Science.* Pp. 112. 5s. net. 1929. (Milford, Ox. Univ. Press.)

Hall, F. G., and Rideal, E. K. *Cambridge Five-Figure Tables.* Pp. viii + 76. 3s. 6d. 1929. (Cam. Univ. Press.)

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Humphrey, D. *Advanced Mathematics : for Students of Physics and Engineering.* Part I. Pp. 1-120. 6s. net. Part II. Pp. 1-175. 7s. 6d. net. In one volume, 12s. 6d. net. 1929. (Milford, Ox. Univ. Press.)

Ives, H. C. *Seven Place Natural Trigonometrical Functions.* Pp. 222. 12s. 6d. net. 1929. (Wiley ; Chapman and Hall.)

Jllsung, F. *Die Descente-Methode für die allgemeine Lösung des Fermat-Problems.* Pp. 16. MK. 1.50. 1930. Becker, Würzburg.

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Krishnaswami Ayyanger, A. A. *A New Geometry.* Pp. xxvii + 600 + xxx. Re. 2.8. 1928. (Srinivasa Varadachari, Madras.)

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Levi, F. *Geometrische Konfiguration, mit einer Einführung in die Kombinatorische Flächentopologie.* Pp. viii + 310. 1929. 24 RM. (Hirzel, Leipzig, C. 1.)

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Milne, E. A. *The Aims of Mathematical Physics.* Pp. 28. 2s. net. 1929. (Milford, Clarendon Press, Oxford.)

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Potter, F. F., and Larrett, D. *A Junior Practical Geometry.* Pp. v + 135. 2s. 6d. 1930. (Pitman.)

Raaz, F. *Sphärische Trigonometrie für Naturwissenschaft und Technik.* Pp. 67. RM. 6. 1929. (Steinkopff, Dresden.)

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Rothe, R. *Höhere Mathematik.* 3rd Edit. RM. 6. 1930. (Teubner.)

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Sommerville, D. M. Y. *An Introduction to the Geometry of N-Dimensions.* Pp. xvii + 196. 10s. 1929. (Methuen.)

Suggestions on Mathematical Work in Training Colleges. *A Memorandum prepared by a Committee of the Mathematics Section of the Training College Association.* (E. R. Hamilton, B.Sc., Univ. Coll., Bangor.)

Thiry, R. *Eléments de Mathématiques Financières. Opérations à long terme.* Pp. viii + 91. 16 fr. 1930. (Vuibert, Paris.)

Turnbull, H. W. *The Great Mathematicians.* Pp. viii + 128. 2s. 6d. 1929. (Methuen.)

Villat, H. *Leçons sur l'Hydrodynamique.* Pp. iv + 296. 50 fr. 1929. (Gauthier-Villars.)

Abstracts of Scientific and Technical Publications from the Massachusetts Institute of Technology. No. 4. July 1929.

Record of Session 1928-1929. *Yorkshire Branch of the Mathematical Association.* Pp. 19. 1929.

Villat, H. *Leçons sur la Théorie des Tourbillons.* Pp. 300. 65 fr. 1930. (Gauthier-Villars.)

Walker, A., and MacNicol, G. P. *A School Geometry.* Part I. Books I-III. Pp. viii + 256. 3s. 6d. Part II. Books IV-VII. 3s. 6d. Section I, Books IV-V, 2s. Section II, Books VI-VII. 2s. 1930. (Longmans, Green.)

Weatherburn, C. E. *Differential Geometry of Three Dimensions.* Vol. II. Pp. xii + 239. 12s. 6d. net. 1930. (Cam. Univ. Press.)

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Boletin Matematico.

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Bollettino della Unione Matematica Italiana.

Bulletin of the American Mathematical Society.

Bulletin of the Calcutta Mathematical Society.

Contribución al Estudio de las Ciencias Físicas y Matemáticas.

Jahresbericht der Deutschen Mathematiker-Vereinigung.

Japanese Journal of Mathematics.

Journal of the Indian Mathematical Society.

Journal of the London Mathematical Society.

Journal of the Mathematical Association of Japan.

Journal de la Société Physico-Mathématique de Léningrade.

L'Enseignement Mathématique.

Mathematics Teacher.

Memoria. (Univ. Nac. de la Plata.)

Monatshefte für Mathematik und Physik.

Nieuw Archief voor Wiskunde.

Nieuwe Opgaven.

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Proceedings of the Edinburgh Mathematical Society.

Proceedings of the Physico-Mathematical Society of Japan.

Publicaciones de la Facultad de Ciencias Físico-Matemáticas Universidad Nacional de la Plata.

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Revista Matemática Hispano-Americana. (Madrid.)

Revue Semestrielle des Publications Mathématiques.

School Science and Mathematics.

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The Eugenics Review.

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Unterrichtsblätter für Mathematik und Naturwissenschaften.

Wiskundige Opgaven met de Oplossingen.

LONDON BRANCH.

PROGRAMME FOR THE SESSION 1930-1.

All meetings are held at Bedford College, Regent's Park, at 3 p.m.

1930.

Oct. 4th. "Napier's Method of Constructing Logarithms and its Advantages for School Use."—W. C. Fletcher, C.B.

Nov. 8th. "Some Methods of Lightning Calculation."—A. H. Russell, (E. Bristol Central School).

Dec. 6th. Presidential Address: "The Value of Exactness."—Dr. Cyril Norwood.

1931.

Jan. 31st. (1) Annual Business.

(2) Discussion of Members' Topics.

Feb. 28th. "The Correlation of Trigonometry and Geometry in Elementary School Mathematics."—W. J. Dobbs.

Mar. 31st. "Contracted Methods in Arithmetic."—S. Inman.

REPORT OF THE SYDNEY BRANCH FOR 1929.

Connected with this Branch there are now 24 members of the parent Association and 83 members of the Branch.

Two meetings were held during the year; at the first of these the President (Prof. Wellish) reported that the Executive had considered certain proposals made at the previous annual meeting about the naming of theorems and the use of abbreviations in written work. At the same meeting Mr. Thorne gave an interesting address on "Geometrical Calculus."

The second was the annual meeting. Reports from the Hon. Secretaries and the Hon. Treasurer were received and adopted. The office-bearers for 1930 were elected as follows:

President - - - -	Prof. H. S. Carslaw.
Joint Hon. Secs. - - -	Miss E. A. West. Mr. H. J. Meldrum.
Hon. Treas. - - - - -	Mr. A. L. Nairn.

A long and interesting discussion took place upon the Intermediate Certificate and Leaving Certificate Examination papers for 1929. The discussion revealed, among other things, the impossibility of the Leaving Certificate Examination serving adequately two different purposes—that of testing the ordinary pupil's work during a school course of study, and that of satisfying university requirements for matriculation.

A paper was to have been read by Mr. Meldrum on "Standardised Tests," but was held over because of the lateness of the hour. At this meeting a very fine book display was given, several firms of publishers co-operating to make the display a success.

H. J. Meldrum, E. A. West, } Joint Hon. Secs.

THE MIDLAND BRANCH.

The Annual General Meeting was held at Birmingham on the 21st March, 1930. Mr. C. T. Preece, of Birmingham University, was elected chairman for the ensuing session. After the election of other officers, short papers were given by Messrs. A. H. Hinckley, P. W. Bates, and C. H. Richards (the retiring chairman).

During the year the Branch has had papers from the following:

Mar. 8th, 1929. Prof. G. N. Watson, of Birmingham, on "Ramanujan."

Oct. 25th, „ Prof. W. P. Milne, of Leeds, on "The Equation viewed as an Historical Romance."

Dec. 7th, „ G. St. L. Carson, H.M.I., on "Addition, Subtraction, Multiplication and Division."

Feb. 7th, 1930. W. C. Fletcher, late H.M.I., on "Perspective Drawing."

The Branch consists of about 25 members and 17 associates, but it is hoped to increase these numbers by a recruiting campaign at the beginning of the Autumn Term.

A. Jackson, Hon. Sec.

PERSONAL NOTES, Etc.

At Oxford the Junior Mathematical Scholarship has been awarded to Mr. E. S. Jackson, Scholar of C.C.C., and the Mathematical Exhibition to Mr. A. G. Walker, Scholar of Balliol.

The Third International Congress of Technical Mechanics will be held at Stockholm, 24th-29th Aug. 1930. The Secretary-General is Prof. W. Weibull, Ecole technique et supérieure, Valhallavägen, Stockholm, Sweden.

The Sheepshanks Exhibition for Proficiency in Astronomy has been awarded to Mr. R. D. H. Jones, Scholar of Gonville and Caius, Cambridge.

THE GENERAL TEACHING COMMITTEE.

A Report on Mathematics in Entrance Scholarship Examinations in Public Schools was published by the Association in 1926. In order that the General Teaching Committee may consider how far the recommendations contained in that report are approved (and acted on) by the Public Schools, copies of papers recently set in scholarship examinations have been collected. Thanks are due to many headmasters and senior mathematical masters for supplying these papers.

Chairman C. O. Tuckey, M.A.

Hon. Sec. A. Robson, M.A., Marlborough College, Wilts.

BOOKS.

The Clarendon Press has issued (April, 1930) Vol. I, No. 1, of *The Quarterly Journal of Mathematics (Oxford Series)*, the successor of the old *Quarterly Journal* and of *The Messenger of Mathematics*. It is to be published in March, June, September, and December, at 7s. 6d. a single number, or at an annual subscription of 27s. 6d. post free.

The Contents of the first number are as follows: *The Motion of a Fluid in a Field of Radiation*. E. A. Milne. *Some Problems connected with Fourier's Work on Transcendental Equations*. G. Pólya. *Electrical Notes*. F. B. Pidduck. *The Zeros of certain Integral Functions*. Miss M. L. Cartwright. *A Generalization of the Quadratic Differential Form*. O. Veblen.

There is little doubt that the new venture will prove worthy of its predecessors. The Editors are Messrs. T. W. Chaundy, W. L. Ferrar, and E. G. C. Poole.

Archibald, R. C. *Mathematics before the Greeks*. Pp. 12. Reprint from *Science*, Jan. 31, 1930.

Batten, T. C., and Brown, M. W. *A School Algebra*. Part I. Pp. viii+198. 3s. 1930. (Murray.)

Breslich, E. R. *The Teaching of Mathematics in Secondary Schools*. Vol. I. *Technique*. Pp. vii+239. 9s. net. 1930. (Univ. Chicago Press, per Camb. Univ. Press.)

Carslaw, H. S. *Plane Trigonometry*. 3rd Edit. Pp. xviii+329+xii. 5s. 1930. (Macmillan.)

Camichel, C. *Leçons sur les Conduites*. Pp. 101. 30 fr. 1930. (Gauthier-Villars.)

Dehn, E. *Algebraic Equations. An Introduction to the Theories of Lagrange and Galois*. Pp. xi+208. 21s. net. 1930. (Columbia Univ. Press, per Mr. Milford or Ox. Univ. Press.)

Durell, C. V., and Fawdry, R. C. *A Concise Arithmetic*. Pp. xii+243+xxvii. 2s. 9d. with Answers; 2s. 6d. without. 1930. (Bell & Sons.)

Durell, C. V., and A. Robson. *Advanced Trigonometry*. Pp. viii+335. 8s. 6d. 1930. (Bell & Sons.)

y Fernández, M. L. *Los Fundamentos de la Aritmética*. Pp. 38. n.p. 1930. (Casas y Mercado, Matanzas.)

Fisher, R. A. *The Genetical Theory of Natural Selection*. Pp. xiv+272. 7s. 6d. net. 1930. (Milford: Clarendon Press.)

Ibbetson, W. S. *Preliminary Mathematics for Engineers*. Pp. xi+152. 3s. 6d. 1930. (Pitman & Sons.)

Jackson, Dunham. *The Theory of Approximation*. (Am. Math. Soc. Colloquium Publications XI.) Pp. viii+178. n.p. 1930. (Am. Math. Soc., 501 West 116th St., New York.)

Papelier, G. *Eléments de Trigonométrie Sphérique*. Pp. 165. 1930. 20 fr. (Vuibert.)

Phillips, E. G. *A Course of Analysis*. Pp. viii+361. 16s. net. 1930. (Camb. Univ. Press.)

Potter, F. F., and Larrett, D. *School Certificate Geometry*. Pp. viii+200. 3s. 6d. 1930. (Pitman.)

Polthof, B. *Die anschauliche Natur der geometrischen Grundbegriffe*. Pp. 28. RM. 1. 1930. (Aschendorffsche Verlag. Munster.)

Sanford, V. *A Short History of Mathematics*. Pp. xii+402. \$3·25. 1930. (Houghton Mifflin Co.)

Wisdom, A. *Arithmetical Dictation*. Book VII. Pp. vii+52. 1s. 6d. 1930. (University of London Press.)

Young, J. W. *Projective Geometry*. [Carus Monographs IV.] Pp. ix+184. \$2. 1930. (Open Court.)

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Bulletin of the American Mathematical Society.

Bulletin of the Calcutta Mathematical Society.

Contribución al Estudio de las Ciencias Físicas y Matemáticas.

Jahresbericht der Deutschen Mathematiker-Vereinigung.

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Journal of the Mathematical Association of Japan.

Journal de la Société Physico-Mathématique de Léningrade.

L'Enseignement Mathématique.

Mathematics Teacher.

Memoria. (Univ. Nac. de la Plata.)

Monatshefte für Mathematik und Physik.

Nieuw Archief voor Wiskunde.

Nieuwe Opgaven.

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Sitzungsberichte der Berliner Mathematischen Gesellschaft.

The Eugenics Review.

The Half-Yearly Journal of the Mysore University.

Unterrichtsblätter für Mathematik und Naturwissenschaften.

Wiskundige Opgaven met de Oplossingen.

REPORTS OF BRANCHES.

LIVERPOOL MATHEMATICAL SOCIETY.

Two meetings of the Society were held at the University of Liverpool in the Easter Term.

At the first meeting, held on February 3, Mr. R. O. Street, of Liverpool University, opened a discussion on "Mathematics in the Higher School Certificate Examination".

At the second meeting, held on March 10, two short papers were read, (i) "The Development of Arithmetic", by Miss M. Ralph, of the Queen Mary High School, Liverpool, (ii) "Advanced Course Mathematics", by Mr. R. Baldwin, of Wallasey Grammar School.

There was an attendance of about 30 members at each meeting.

The annual general meeting was held on May 13, when Prof. G. H. Hardy lectured on "Prime Numbers". About 70 members and visitors were present on this occasion.

Prof. E. C. Titchmarsh, of Liverpool University, was elected President of the Society for the Session 1930-1931.

R. BALDWIN, Hon. Sec.

NORTH-EASTERN BRANCH.

A MEETING of the Branch was held on March 15, at Armstrong College, Newcastle-upon-Tyne. The President (Mr. J. Strachan, H.M.I.), took the chair at 3.15 p.m., 28 members and associates being present.

Mr. E. R. Verity read a paper on "The Teaching of Mathematics in Technical Colleges with special reference to Graphical Methods". He stated that technical education is a training in the art of utilising knowledge for the manufacture of material products and that it did not lack the elements of true education: culture, etc. In the Junior Technical School are boys between the ages of 12+ and 16 years, and the freedom from external examinations leaves the teachers better facilities than in other schools for associating mathematics with other subjects.

In the Technical College, day students are from 17+ to 22 years studying for a Diploma or a Degree. There are also some part-time, usually evening, students.

The teaching should be as general as possible in methods and as concrete as possible in final results, with the hope of securing the maximum of interest and the ability of the student to carry on, after his college days, continued development of his knowledge and skill in his subject.

Accurate numerical work is absolutely vital, yet students are not to attempt to give answers with more significant figures than are justified by the data. Graphical work plays an important part in the work. Algebra is regarded as a Calculus of Functions and Arithmetic as a Calculus of Values. Other subject matter studied: Laws represented by given curves, turning points, inflexions, rates of change, integrals, mean values, etc., in many settings and applied to many kinds of physical problems.

The scheme includes Descriptive Geometry, *i.e.* the science of solving by Plane Geometry problems connected with the Geometry of Space, chiefly by the method of Monge.

The lecture concluded with a number of graphical illustrations.

J. W. BROOKS, Hon. Sec.

THE VICTORIA BRANCH.

REPORT FOR THE SESSION 1929.

Office Bearers.

Honorary President: PROFESSOR NANSON. *President*: PROFESSOR CHERRY. *Vice-Presidents*: PROFESSOR MICHELL; DR. J. M. BALDWIN; MISS K. GILMAN JONES; MR. D. K. PICKEN; MR. M. S. SHARMAN. *Secretaries*: MR. R. J. A. BARNARD; Mr. J. L. GRIFFITHS. *Treasurer*: MR. F. W. CAMPBELL. *Other Members of Committee*: MISS J. T. FLYNN; MISS WADDELL.

FOUR meetings were held during the year, as follows:

April 16. A very large attendance of about 60 members and visitors was present to hear an address by Professor Cherry on "Relations between Geometry and Algebra". The lecturer explained diagrammatic methods of representing multiplication and division of irrational numbers, and a graphical method of finding approximations to any rational or irrational fraction, showing its connection with the algebraic method of finding convergents.

June 18. Mr. Picken gave a paper on the use of homogeneous coordinates in geometry. He dealt with the importance of homogeneity in quite elementary work in analytical geometry.

July 16. Mr. Barnard gave a paper on "The Two Body and Three Body Problems in Astronomy." He outlined the methods of calculating orbits of a double star from telescopic and from spectroscopic data and referred to cases of the three body problem in the solar system and in triple stars.

September 17. The meeting was devoted to short papers.

Mr. Barnard gave a note on the factorization of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$.

Mr. Belz gave a description of ruled probability paper and its uses.

Mr. Barclay showed some interesting examples of the use of probability paper in meteorological and other applications.

Professor Cherry showed and explained a model of a one-sided polyhedron.

Instead of a fifth meeting the Committee invited members to afternoon tea at Yarra House, Church of England Girls' Grammar School, which was kindly put at their disposal by Miss K. Gilman Jones. This meeting was generally felt to be a very successful innovation for the Association.

R. J. A. BARNARD, Hon. Sec.

REVISTA MATEMÁTICA HISPANO-AMERICANA.

THE Sociedad Matemática Española has been kind enough to supply several numbers which were missing, and the Library has now a set complete from the beginning in 1919.

STUDIA MATHEMATICA.

A MATHEMATICAL periodical of Western Europe may become in fact associated specially with one or two fields of mathematics, but it has been left to Poland to establish periodicals deliberately restricted. First came *Fundamenta Mathematicae*, devoted to the borderland between mathematics and symbolic logic, and now of *Studia Mathematica*, published at Lwow, we are told "La Rédaction poursuit le but de grouper autour de ce journal les recherches concernant l'analyse fonctionnelle et tout ce qui s'y rattache". The editors are S. Banach and H. Steinhaus, and the first volume, of 256 pages, contains twelve papers, all of a high standard of interest.

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W. G. Borchardt. *Geometry Test Papers.* Pp. 88, viii. Wraps. 1s. 1930. (Rivingtons.)

J. I. Craig. *Elements of Analytical Geometry.* I: Straight Line and Circle. Pp. xiv, 416. 12s. 6d. 1930. (Macmillan.)

W. M. Deans. *Examples in Elementary Algebra.* Pp. 48. Wraps. 1s. 1930. (Blackie.)

L. E. Dickson. *Studies in the Theory of Numbers.* Pp. x, 230. 18s. 1930. (Univ. of Chicago Press; C. U. P.)

P. A. M. Dirac. *The Principles of Quantum Mechanics.* Pp. x, 258. 17s. 6d. 1930. (Clarendon.)

J. Dougall. *Test Papers in Algebra and Geometry.* (For Public School Entrance Scholarship Examinations.) Pp. 40. Wraps. 1s. 1930. (Blackie.)

A. Dresden. *Solid Analytical Geometry and Determinants.* Pp. x, 310. 15s. 1930. (Wiley and Chapman & Hall.)

C. V. Durell. *A New Algebra for Schools,* Parts I-II with Appendix. Pp. x, 328, xxiv, xxiv, xvi. 4s. 6d. (Without Appendix, 3s. 6d.; without answers, 4s. and 3s.) 1930. (Bell.)

T. Fort. *Infinite Series.* Pp. iv, 254. 20s. 1930. (Clarendon.)

D. Hilbert. *Grundlagen der Geometrie.* 7 Aufl. Pp. viii, 326. Rm. 18. 1930. Wissenschaft und Hypothese, 7. (Teubner.)

D. Humphrey. *Intermediate Mechanics: Dynamics.* Pp. xii, 382. 10s. 6d. 1930. Longmans' Modern Mathematical Series. (Longmans, Green.)

B. C. Molony. *A Numerical Trigonometry.* Pp. 216. 3s. 1930. (Arnold.)

F. R. Moulton. *Differential Equations.* Pp. xvi, 396. 24s. 1930. (The Macmillan Co.)

P. Painlevé. *Leçons sur la Résistance des Fluides non visqueux.* I. Pp. viii, 184. 40 fr. 1930. (Gauthier-Villars.)

J. Rey Pastor. *Teoría Geométrica de la Polaridad.* Pp. viii, 294. Pesetas 20. 1929. (Rev. Mat. Hispano-Americana, Madrid.)

K. Reidemeister. *Grundlagen der Geometrie.* Pp. x, 148. Rm. 11. (geb. 12.60.) 1930. Grundlehren der math. Wiss., 32. (Springer.)

L. Silberstein. *The Size of the Universe.* Pp. viii, 216. 10s. 1930. (Oxford.)

G. W. Spriggs. *Introductory School Algebra.* Pp. xii, 136. 2s. 6d. 1930. (Pitmans.)

G. W. Spriggs. *School Certificate Algebra.* Pp. xviii, 326. 4s. 1930. (Pitmans.)

E. C. Titchmarsh. *The Zeta-Function of Riemann.* Pp. viii, 104. 6s. 6d. 1930. Cambridge Tracts, 26. (C. U. P.)

A. E. Tweedy. *Senior Geometry.* Part I. Pp. x, 236. 2s. 9d. 1930. (Dent.)

J. H. Wells. *Thirty Tests in Elementary Mathematics.* Pp. 44. Wraps. 8d. 1930. (Harrap.)

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American Mathematical Monthly. **37**: 5, 6.

Anales de la Sociedad Científica Argentina. **109**: 4.

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Annals of Mathematics. **31**: 2, 3.

Anuario (Univ. Nac. de la Plata).

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Bollettino della Unione Matematica Italiana. **9**: 3.

Bulletin of the American Mathematical Society. **36**: 5, 6.

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Half-Yearly Journal of the Mysore University. **4**: 1.

Jahresbericht der Deutschen Mathematiker-Vereinigung. **39**: 5-8.

Japanese Journal of Mathematics. **7**: 1.

Journal of the Indian Mathematical Society. **18**: 8, 9.

Journal of the London Mathematical Society. **5**: 3.

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Periodico di Matematiche. Ser. 4. **10**: 4.

Proceedings of the Edinburgh Mathematical Society. Ser. 2. **2**: 2.

Proceedings of the Physico-Mathematical Society of Japan. Ser. 3. **12**: 5, 6.

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Revista Matemática Hispano-Americana (Madrid). Ser. 2. **5**: 2-3.

Revue Semestrielle des Publications Mathématiques.

School Science and Mathematics. **30**: 6, 7.

Sitzungsberichte der Berliner Mathematischen Gesellschaft. **29**: 1.

Unterrichtsblätter für Mathematik und Naturwissenschaften. **36**: 6, 7, 8.

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P. Barlow. *Tables of Squares, Cubes, Square Roots, Cube Roots, and Reciprocals.* Third edition, edited by L. J. Comrie. Pp. xii, 208. 7s. 6d. 1930. (Spon.)

L. Brand. *Vectorial Mechanics.* Pp. xviii, 544. 25s. 1930. (Wiley; Chapman & Hall.)

E. T. Chisnell. *By Graph to Calculus.* Pp. xii, 86. 2s. (Or in two parts, 10d., 1s. 3d.) 1930. (Harrap.)

E. Dehn. *Algebraic Charts.* 6 in folder. 3s. 6d. 1930. (Oxford Univ. Press.)

C. V. Durell and **A. Robson.** *Key to Advanced Trigonometry.* Pp. 380. 15s. 1930. (Bell.)

M. Ezekiel. *Methods of Correlation Analysis.* Pp. xiv, 428. 22s. 6d. 1930. (Wiley; Chapman & Hall.)

H. G. Forder. *A School Geometry.* Pp. x, 260. 4s. 6d. 1930. (Camb. Univ. Press.)

A. R. Forsyth. *Geometry of Four Dimensions.* Two vols. Pp. xxx, 468; xii, 520. 75s. 1930. (Camb. Univ. Press.)

A. Fraenkel. *Georg Cantor.* Pp. 78. Wraps. Rm. 4.50. 1930. Reprint from *Jbrt. d. deutschen Math. Verein.*, vol. 39. (Teubner.)

H. Hasse. *Bericht über neuere Untersuchungen und Probleme aus der Theorie der algebraischen Zahlkörper.* Iu, Ia, II. Pp. 134, iv + 204. Wraps. Rm. 7.40, 11.80. Reprinted from *Jbrt. d. deutschen Math.-Verein.*, vols. 35, 36. 1930. (Teubner.)

J. O. Hassler and **R. R. Smith.** *The Teaching of Secondary Mathematics.* Pp. xii, 406. 10s. 6d. 1930. (The Macmillan Co.)

K. Hayashi. *Fünfstellige Funktionentafeln.* Pp. viii, 176. Rm. 28 (geb. Rm. 30). 1930. (Springer.)

K. Hayashi. *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen.* Pp. vi, 126. Rm. 24 (geb. Rm. 26). 1930. (Springer.)

W. Heisenberg. *The Physical Principles of the Quantum Theory.* Translated by C. Eckart and F. C. Hoyt. Pp. xii, 186. 8s. 6d. 1930. (Univ. of Chicago Press; Camb. Univ. Press.)

A. F. van der Heyden. *Elementary Trigonometry.* Pp. viii, 164, xiv. 2s. 6d. 1930. (Rivington.)

T. Hodgson. *Applied Mathematics for Engineers.* Vol. I. Graphical Statics. Pp. viii, 184. 9s. 6d. Vol. II. Dynamics and Calculus. Pp. viii, 294. 13s. 6d. 1930. (Chapman & Hall.)

J. H. Jeans. *The Mysterious Universe.* Pp. x, 154. 3s. 6d. 1930. (Camb. Univ. Press.)

R. de L. Kronig. *Band Spectra and Molecular Structure.* Pp. x, 164. 10s. 6d. 1930. (Camb. Univ. Press.)

E. A. G. Lamborn. *Reason in Arithmetic.* Pp. iv, 140. 3s. 6d. 1930. (Clarendon Press.)

S. Lefschetz. *Topology.* Pp. x, 410. $4.50. 1930. American Math. Soc. Colloquium Publications, 12. (Amer. Math. Soc.)

E. H. Lockwood. *A Revision Arithmetic.* Pp. viii, 176, viii. 2s. 6d. (With or without Answers.) 1930. (Longmans, Green.)

T. Muir. *Contributions to the History of Determinants*: 1900-1920. Pp. xxiv, 408. 30s. 1930. (Blackie.)

J. Peters. *Siebenstellige Werte der Trigonometrischen Funktionen.* Pp. 376. Wraps. Rm. 18 (geb. Rm. 21). Reprint. (Teubner.)

H. Rademacher and **O. Toeplitz.** *Von Zahlen und Figuren.* Pp. vi, 164. Rm. 9.60. 1930. (Springer.)

G. Rosmanith. *Mathematische Statistik der Personenversicherung.* Pp. vi, 142. Rm. 8. 1930. Sammlung math.-phys. Lehrbücher, 28. (Teubner.)

C. W. Saurin. *The Wide Outlook Arithmetics.* II. Pp. 80. Limp Wraps. 1s. 1930. (Blackie.)

W. F. F. Shearcroft and C. N. Lewis. *School Certificate Mechanics and Hydrostatics.* Pp. viii, 376. 4s. 6d. 1930. (Pitman.)

A. E. Tweedy. *Senior Geometry.* Part II. Pp. x, 180. With Answers. 2s. 3d. 1930. (Dent.)

V. Volterra. *Theory of Functionals and of Integral and Integro-Differential Equations.* Edited by L. Fantappiè. Translation by M. Long. Pp. xiv, 226. 25s. 1930. (Blackie.)

B. L. van der Waerden. *Moderne Algebra.* I. Pp. viii, 244. Rm. 15.60 (geb. 17.20). 1930. Grundlehren der math. Wiss., 33. (Springer.)

J. J. Walton. *School Certificate Trigonometry (with Mensuration).* Pp. viii, 196. 4s. (without Answers, 3s. 6d.). 1930. (Pitman.)

J. J. Walton. *Test Papers in Trigonometry and Calculus.* Pp. iv, 104. Wraps. 2s. 6d. 1930. (Pitman.)

O. Weinberger. *Mathematische Volkswirtschaftslehre.* Pp. xiv, 242. Rm. 18. 1930. (Teubner.)

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School Science and Mathematics. 30 : 8.
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Unterrichtsblätter für Mathematik und Naturwissenschaften. 36 : 9, 10, 11.
Wiskundige Opgaven met de Oplossingen.

LONDON BRANCH.

A MEETING was held on Saturday, October 4, at Bedford College, with Mr. Katz in the chair. There were 84 present.

It was announced that Mr. Boon had resigned the office of Secretary to the Branch, and that Mr. Daltry had been elected in his place. The thanks of the Branch were accorded to Mr. Boon for his outstanding work as Secretary.

Mr. W. C. Fletcher read a paper on "Napier's Construction of Logarithms and its Advantages for School Use". The widespread introduction to logarithms by graphs and indices held up the practical use of tables in lower forms, and was not now fundamental in advanced theoretical work. He preferred an exploration of the actual tables followed by the construction of tables built up by simple multiplication from two geometric progressions. Napier's original method of interpolation could be used with advantage since it helped with the idea of a limit and with the integration of x^{-1}. Applications of Napier's method to $\lim_{n\to\infty} (1+1/n)^n$ and to the Logarithmic Series were shown; a discussion followed.

C. T. DALTRY, Hon. Sec.

LONDON BRANCH.

A MEETING was held on November 8, to hear an address by Mr. A. H. Russell, of the East Bristol Central School, on "Some Methods of Lightning Calculation". 71 members and associates were present. Mr. Russell emphasised two fundamental principles, addition by "double columns", and multiplication from the right. Using these and simple algebraic ideas it was possible to extend mental multiplications, and squarings to 99×99. Certain division sums could also be done mentally simply by considering remainders. In money calculations Mr. Russell dealt with Simple Interest, Simple and Compound Practice and Decimalisation of Money.

Before the address it was announced that in consequence of the memorandum recently submitted to certain examining boards, the committee had

been invited to submit a syllabus in "Numerical Trigonometry" for School Certificate Examination for the consideration of the Oxford and Cambridge Joint Board. C. T. Daltry, Hon. Sec.

MANCHESTER AND DISTRICT BRANCH.

Two meetings of the Branch have been held at Manchester High School during the autumn term, Miss W. Garner presiding on both occasions. Seven new members have joined the Branch, and the average attendance has been thirty-nine.

On Tuesday, October 7, Miss E. Willis spoke on "The Teaching of Partial Fractions" for which she suggested an interesting method depending on Limits. The second part of this meeting was devoted to a discussion of the papers set in 1930 for the Higher Certificate and the School Certificate Examinations of the N.U.J.M.B. It was resolved that the criticisms and suggestions of the Branch should be forwarded to the Joint Matriculation Board.

At the meeting on Tuesday, November 18, Mr. C. O. Tuckey spoke on "The Teaching of Arithmetic: Some points from the forthcoming Report of the Boys' Schools Committee". Mr. Tuckey said that Arithmetic, though it is the subject which the mathematical teacher tends to despise, is in point of time the major subject of the average child's mathematical career, and in the case of the weaker pupils it is the only mathematical subject in which the child can feel that in some part mastery has been obtained. At the end of Mr. Tuckey's lecture there was considerable discussion, which centred particularly round methods of working decimals. In proposing a vote of thanks to Mr. Tuckey, Mr. Trevor Dennis expressed also the gratitude of teachers to those committees of the Mathematical Association which give their time to the preparation of reports on teaching.

During next term the Branch hopes to hold two meetings. At the first, on Friday, January 30, Miss F. L. Baugh will be the speaker; at the joint meeting with the University Mathematical Society, on Wednesday, February 25, Professor F. J. M. Stratton will speak on "Some Eclipse Problems".

M. O. Stephens, Hon. Sec.

YORKSHIRE BRANCH.

A meeting of the Yorkshire Branch was held on Saturday, November 8, 1930.

The following elections were made: Vice-Presidents, Miss M. E. Bowman, Miss L. Watson. Treasurer, Miss I. M. Mathews. The retiring Treasurer was heartily thanked by the Branch for her excellent services in the past three years.

A paper on "The Isoperimetrical Problem" was read by Mr. R. M. Gabriel, Reader in Pure Mathematics in the University of Leeds. This paper was remarkable for its simplicity and the small amount of geometrical knowledge demanded.

A Presidential address was delivered by Mr. A. B. Oldfield, in which he suggested that elementary vectors might without difficulty find a place in the sixth form curriculum.

UNIVERSITY OF LA PLATA.

The Comisión de Publicaciones has been kind enough to supply certain of the numbers missing from our set, and we have now the *Contribución al Estudio de las Ciencias Físicas y Matemáticas* complete except for one part (No. 28; Ser. Técnica, vol. i. ent. 5) of which no copy can be found for us.

JOURNALS RECEIVED.

When no number is attached, no part has been received since a previous acknowledgment.

Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität.

American Journal of Mathematics.

American Mathematical Monthly. 38: 1, 2.

Anales de la Sociedad Científica Argentina. 110: 6; **111:** 1.

Annales de la Société Polonaise de Mathématique.

Annals of Mathematics.

Berichte über die Verhandlungen der Akad. der Wiss. zu Leipzig: Math.-Phys. Klasse.

Boletin Matematico. 3: 11.

Boletin Matematico Elemental.

Boletín del Seminario Matemático Argentino.

Bollettino della Unione Matematica Italiana. 10: 1.

Bulletin of the American Mathematical Society. 37: 1, 2.

Bulletin of the Calcutta Mathematical Society. 22: 2-3.

Contribución al Estudio de las Ciencias Físicas y Matemáticas.

L'Enseignement Mathématique.

Half-Yearly Journal of the Mysore University.

Jahresbericht der Deutschen Mathematiker-Vereinigung. 40: 1-4.

Japanese Journal of Mathematics. 7: 3.

Journal of the Indian Mathematical Society. 18: 12.

Journal of the London Mathematical Society. 6: 1.

Journal of the Mathematical Association of Japan. 12: 6; **13:** 1.

Journal de la Société Physico-Mathématique de Léningrade.

Mathematical Notes.

Mathematics Teacher. 24: 2.

Memoria (Univ. Nac. de la Plata).

Monatshefte für Mathematik und Physik.

Nieuw Archief voor Wiskunde.

Nieuwe Opgaven.

Periodico di Matematiche. Ser. 4. 11: 1.

Proceedings of the Edinburgh Mathematical Society. Ser. 2. 2: 3.

Proceedings of the Physico-Mathematical Society of Japan. Ser. 3. 12: 11; **13:** 1.

Publicaciones de la Facultad de Ciencias Fisico-Matemáticas Universidad Nacional de la Plata.

Publications de la Faculté des Sciences de Masaryk. 126, 127.

Quarterly Journal of Mathematics. Oxford Series. 1: 3, 4.

Revista de Ciencias (Peru).

Revista Matemática Hispano-Americana (Madrid). Ser. 2. 5: 9-10.

Revue Semestrielle des Publications Mathématiques. 35.

School Science and Mathematics. 31: 2, 3.

Sitzungsberichte der Berliner Mathematischen Gesellschaft. 29: 2.

Studia Mathematica.

Unterrichtsblätter für Mathematik und Naturwissenschaften. 37: 2, 3.

Wiskundige Opgaven met de Oplossingen.

BOOKS RECEIVED.

W. S. Beard. *Bell's Practical Modern Arithmetics for Senior Classes.* Pt. III, pp. v + 74. Paper, 10d. Cloth, 1s. 1931. (Bell & Sons.)

L. Bieberbach. *Lehrbuch der Funktionentheorie II.* (2 aufl., pp. vi + 370.) Rm. 20. 1931. (Teubner.)

C. V. Durell. *A New Algebra for Schools.* Pt. III, pp. xiii + 198 + xxxviii. Without appendix: with answers 3s., without answers 2s. 6d. With appendix: with answers 3s. 6d., without answers 3s. 1931. (Bell & Sons.)

C. V. Durell. *Shorter Geometry.* Pp. iv + 156. 3s. 1931. (Bell & Sons.)

H. Freeman. *An Elementary Treatise on Actuarial Mathematics.* Pp. x + 399. 25s. 1931. (Cambridge Univ. Press.)

J. W. M. Gunn. *Trigonometry Test Papers.* Pp. xii + 92. 1s. 1931. (Rivingtons.)

W. R. Hamilton. *Mathematical Papers. Vol. I. Geometrical Optics.* Pp. xxviii + 534. 50s. 1931. (Cambridge Univ. Press.)

W. H. Hewitt. *School Certificate Sound.* Pp. vi + 177. 3s. 6d. 1931. (Pitman.)

L. M. Milne-Thomson and L. J. Comrie. *Standard Four-Figure Mathematical Tables.* (Edition B.) Pp. xvi + 245. 10s. 6d. 1931. (Macmillan.)

J. B. Scarborough. *Numerical Mathematical Analysis.* Pp. xiv + 416. 25s. 1930. (Johns Hopkins Press: Humphrey Milford.)

T. Y. Thomas. *The Elementary Theory of Tensors.* Pp. ix + 122. 10s. 1931. (McGraw-Hill.)

LONDON BRANCH.

THE Annual Business Meeting of the Branch was held on 31st January, 1931, at 3 p.m., at Bedford College. Mr. Katz was in the chair, and there were present 61 members and 3 visitors.

The Secretary announced that membership had increased during the past year by 31 full and 13 associate members, and now stood at 169 full and 95 associate members.

It was announced that Professor Roberts desired to resign from the office of Treasurer, a post he had held since the formation of the Branch twenty-one years ago.

A letter from the University of London was read stating that, after consideration of the memorandum on Trigonometry submitted by the London Branch, the Senate had resolved "That in and after 1933 the Arithmetic paper in the General School Examination contain some alternative questions in numerical trigonometry".

The executive officers for the year 1931 were elected as follows: President, J. N. W. Sullivan; Chairman, A. W. Siddons; Vice-Chairmen, F. C. Boon, Miss L. Zelensky; Representative on the Mathematical Association Council, J. Katz; Treasurer, W. G. Bickley; Secretaries, C. T. Daltry, Miss F. A. Yeldham.

The meeting then proceeded to discuss the following topics: "A New Method for Multiplication and Division of Decimals" (Dr. Sheppard); "Some Uses of the Theory of Equations" (Dr. Bickley); "Treatment of Proportion in Geometry" (Mr. Kearney); "Converse Propositions in Elementary Geometry" (Mr. E. P. C. Smith).

A meeting was held at Bedford College on 28th February, 1931. Mr. Siddons was in the chair, and there were present 52 members and 4 visitors.

Mr. W. J. Dobbs read a paper on "The Correlation of Geometry and Trigonometry in Elementary School Mathematics". This paper is to be printed in full in a forthcoming number of the *Gazette*.

C. T. Daltry, Hon. Sec.

QUEENSLAND BRANCH.

The Annual Meeting of the Queensland Branch for the year 1929-1930 was held on 28th March, 1930, and the following report for the year was presented:

The Annual Meeting for 1928-1929 was held at the University on 22nd March, 1929. The Annual Report and the Balance Sheet were presented and adopted, after which the election of officers for the ensuing year took place. Professor Priestley in his presidential address spoke on "The early history of the Theory of Limits".

During the year three general meetings were held. At the first, on 24th May, Dr. E. F. Simonds read a paper on "The Continuum". The second was held on 9th August, at which Professor R. W. Hawken read a paper on "Mathematics in Engineering". At the third, held on 1st November, Mr. E. W. Jones read a paper on "Interpolation".

The number of members of the Branch is 28, of whom 11 are full members of the Mathematical Association: the total membership shows an increase of three on the figures of the previous year. Copies of the *Gazette* come to hand regularly and are circulated among associate members. The statement of receipts and expenditure reveals a satisfactory financial condition, the balance in hand being £6 8s. 4d.

The attendance at meetings has been satisfactory, and no difficulty is experienced in securing speakers and papers, though in this connection it is the metropolitan members on whom the Branch has to rely. An attempt has been made to secure papers from country members, who so far have been reluctant to provide these.

J. P. McCarthy, Hon. Sec.

SYDNEY BRANCH.

At the Annual Meeting of the Sydney Branch the following motion was carried:

"The Sydney Branch of the Mathematical Association at its Annual Meeting on 5th December, 1930, desires to put on record its sense of the loss the Association has suffered in the death of Mr. W. J. Greenstreet, Editor of the *Mathematical Gazette* for so many years; and requests the Honorary Secretary to forward a copy of this resolution to the Secretary of the Association."

Report for the year 1930.

The Sydney Branch has now 21 members and 90 associates. During the year there were two meetings. At the June meeting Mr. Meldrum gave a paper on "Standardised Tests". A brief survey was made of the whole field of testing school mathematical work; the various purposes to be served by different tests were indicated, the scoring of test papers was discussed, and the distinction was drawn between subjective and objective scoring. A brief outline was given of various attempts that had been made to produce standardised tests for secondary schools.

At the Annual Meeting in December the following officers were elected for 1931: President, Professor H. S. Carslaw; Hon. Secretaries, Miss E. A. West, Mr. H. J. Meldrum; Hon. Treasurer, Mr. A. L. Nairn.

The mathematical papers set at the recent Intermediate and Leaving Certificate Examinations were discussed.

An address was given by the President, entitled " A Talk to Teachers ". First, a brief reference was made to the standard of school work in New South Wales as compared with the work done in the best of the English schools. But the main part of the address was an appeal to teachers to continue the study of some branch of mathematics for its own sake. In this way the teacher's work would remain fresh; and in some measure the tendency to deepen the groove in which he was likely to find himself was prevented. A brief outline was given of some of the directions in which such reading might with advantage be undertaken. Teachers were invited to discuss reading plans with the President at any time.

Throughout the year copies of the *Gazette* were circulated among the associate members. E. A. West, H. J. Meldrum, Hon. Secs.

BUREAU FOR THE SOLUTION OF PROBLEMS.

This is under the direction of Mr. A. S. Gosset Tanner, M.A., Derby School, Derby, to whom all inquiries should be addressed, accompanied by a stamped and addressed envelope for the reply. Applicants, who must be members of the Mathematical Association, should wherever possible state the source of their problems and the names and authors of the text-books on the subject which they possess. As a general rule the questions submitted should not be beyond the standard of University Scholarship Examinations. The names of those sending the questions will not be published.

GLEANINGS: AN APPEAL.

The Editor will be grateful for help in the filling up of odd corners. A precise reference should accompany every quotation.

BOOKS RECEIVED.

Bagi, B. B. *Plane Trigonometry.* Pp. iv + 248. 8 Rs. 8 An. 1931. (Dharwar.)

Ballantine, J. P. *The Macmillan Table Slide Rule.* Pp. 4 + 4 slides + 4 plates. 2s. 3d. 1931. (Macmillan.)

van den Berg, P. J. *Reeksen en Machten van den Scherpen Hoek.* Pp. 39. No price. No date.

Borchardt, W. G. *A First Course in Arithmetic.* Pp. viii + 202 + xxxvi. 2s. 6d. 1931. (Rivingtons.)

Breslich, E. R. *Problems in Teaching Secondary-School Mathematics.* Pp. vii + 348. 13s. 6d. 1931. (Univ. Chicago Press; Camb. Univ. Press.)

Crabb, J. A. *Revision Exercises in Mathematics for General School Examinations. Arithmetic. Algebra. Geometry.* Pp. ix + 115. 2s. 1931. (Macmillan.)

Cugnin, L. *L'Ether Immobile est la grande erreur de la Science.* No price. 1926. (Les presses Universitaires de France.)

Durell, C. V. *Stage A Trigonometry.* Pp. 92. With tables, 1s. 9d. Without tables, 1s. 6d. 1931. (Bell.)

Durell, C. V., and Tuckey, C. O. *Simplified Geometry.* Pp. x + 280 + 24. 4s. Or in three parts, 1s. 6d. each. 1931. (Bell.)

Fawdry, R. C. *A Junior Arithmetic.* Pp. vii + 178 + xl. Without answers, 2s. With answers, 2s. 6d. 1931. (Bell.)

Gheury de Bray, M. E. J. *Elementary Hyperbolics,* Vol. I. *Hyperbolic Functions of real and unreal angles.* Pp. xi + 351. 7s. 6d. Vol. II. *The Applications of Hyperbolic Functions.* Pp. xii + 209. 7s. 6d. 1931. (Crosby Lockwood & Son.)

Hamilton, G. H. *A Companion to Elementary Geometry.* Pp. vi + 87. 2s. 6d. 1931. (Blackie.)

Hodgson, T. *Applied Mathematics for Engineers.* Vol. III. *Differential Equations with Applications.* Pp. viii + 320. 13s. 6d. 1931. (Chapman and Hall.)

Keyser, C. J. *Humanism and Science.* Pp. xxii + 243. 15s. 1931. (Columbia Univ. Press; Humphrey Milford.)

Larrett, D. *A Junior Algebra.* Pp. 258 + Answers. 4s. 6d. 1931. (Harrap.)

Lichtenstein, L. *Vorlesungen über einige klassen nicht linearer Integralgleichungen und Integro-differentialgleichungen.* Pp. x + 164. RM. 16.80. 1931. (Teubner.)

Loria, G. *Storia delle Mathematiche.* Vol. II. *I Secoli XVI e XVII.* Pp. 595. L. 23. 1931. (Soc. Tip.-Edit. Naz., Torino.)

McConnell, A. J. *Applications of the Absolute Differential Calculus.* Pp. xii + 318· 20s. 1931. (Blackie.)

Northrop, F. S. C. *Science and First Principles.* Pp. xiv + 299. 12s. 6d. 1931. (Camb. Univ. Press.)

von Sanden, H. *Darstellende Geometrie.* Pp. viii + 111 + 42. RM. 6.40. 1931. (Teubner.)

Schwerdt, H. *Die Anwendung der Nomographie in der Mathematik.* Pp. vii + 116 + Tables 1-104. RM. 28. 1931. (Springer.)

Temple, G. *An Introduction to Quantum Theory.* Pp. 196. 12s. 6d. 1931. (Williams and Norgate.)

Todhunter, R. *The Institute of Actuaries' Text-Book on Compound Interest and Annuities-Certain.* 3rd Ed. Revised by R. C. Simmonds and T. P. Thompson. Pp. xiv + 262. 17s. 6d. 1931. (Camb. Univ. Press.)

Westaway, F. W. *Craftsmanship in the Teaching of Elementary Mathematics.* Pp. xvi + 665. 15s. 1931. (Blackie.)

Abstracts of Dissertations approved for the Ph.D., M.Sc., and M.Litt. Degrees in the University of Cambridge, 1929-1930. Pp. 125. No price. 1931. (Camb. Univ. Press.)

Special Jubilee Number of the Journal of the Society of Chemical Industry. Pp. 272 + 110. 10s. July 1931.

CARDIFF BRANCH.

The year 1930-1931 has been a particularly successful one for the Cardiff Branch. As a result of a " recruiting campaign " on behalf of the Association, the number of full members—that is, members of the branch who are also members of the Association—has increased from 5 to 22. There are, in addition, 17 associate members and a large number of student members.

Five meetings were held during the year.

At the Annual General Meeting on 20th October, Dr. W. Parry-Morgan was elected President for the current year, and the retiring President (Mr. V. W. E. Evans) gave an address on " Cubic Curves." On 17th November we were fortunate in securing as speaker Mr. A. W. Siddons, who spoke to a large audience on " The First Few Years of Geometry in the Secondary School." On 15th December, Dr. D. G. Taylor spoke on " The Equation of the Fifth Degree," illustrating his address with some elegant models and diagrams; while on 2nd February Prof. G. H. Livens opened a discussion on the recent report of the Association on " The Teaching of Mechanics in Secondary Schools." The session concluded on 2nd March with a paper by Miss I. Viney on " The Use of Vectors in Teaching."

H. A. Hayden, Hon. Sec.

LIVERPOOL BRANCH.

The Annual Meeting of the Liverpool Branch for the year 1930-1931 was held at the University on Monday, 18th May, 1931, when Prof. E. T. Whittaker gave an address on " The Nature of Physical Space." About sixty members and visitors were present at this meeting.

Four other meetings of the Branch have been held during the year, at which the following papers were read:

Nov. 18, 1930.—" Irrational Numbers," by Prof. E. C. Titchmarsh.

Dec. 5, 1930.—" Certain difficulties in the teaching of School Mathematics," by Mr. J. Smith.

Feb. 16, 1931.—" Wave Mechanics," by Prof. J. Rice.

March 16, 1931.—" Earthquakes," by Mr. R. O. Street.

There was an average attendance of thirty at these meetings.

The number of members of the Liverpool Branch is now about thirty-five, of whom thirteen are full members of the Mathematical Association.

R. Baldwin, Hon. Sec.

ERRATA TO NEWTON MEMORIAL VOLUME.

The following corrections to Mr. D. C. Fraser's article on Interpolation in the *Isaac Newton* Memorial Volume should be noted:

p. 66, l. 6: read " . . . is equal to the *adjusted* difference of the inclined diagonals, namely, $(FC-BE)/(b-a)$."

p. 67, l. 19: read " . . . is equivalent to the *adjusted* difference of the inclined diagonals."

p. 67, ll. 23, 24: read

$$\text{“}\left.\begin{matrix} B\,.\,U_c \\ -C\,.\,U_b \end{matrix}\right\}\Big/(c-b)+\left\{\begin{matrix} \dfrac{ABC}{1\,.\,2\,.\,3}\Delta^2(bcd) \\ -\dfrac{BCD}{1\,.\,2\,.\,3}\Delta^2(abc) \end{matrix}\right\}\Big/\left(\frac{d-a}{3}\right)+\text{etc.,”}$$

p. 67, ll. 27, 28: read

$$\text{“}\left.\begin{matrix} U_c+\dfrac{B\,.\,C}{1\,.\,2}\Delta(cd) \\ -\dfrac{C\,.\,D}{1\,.\,2}\Delta(bc) \end{matrix}\right\}\Big/\left(\frac{d-b}{2}\right)\begin{matrix} +\dfrac{ABCD}{1\,.\,2\,.\,3\,.\,4}\Delta^3(bcd)x \\ -\text{etc.} \end{matrix}\Bigg\}\Big/\left(\frac{x-a}{4}\right).\text{”}$$

JOURNALS RECEIVED.

When no number is attached, no part has been received since a previous acknowledgment.

Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität.

Abstracts . . . from the Massachusetts Institute of Technology. 7.

American Journal of Mathematics. 53 : 2, 3.

American Mathematical Monthly. 38 : 3, 4, 5, 6.

Anales de la Sociedad Científica Argentina. 111 : 2, 3, 4, 5, 6.

Annales de la Société Polonaise de Mathématique.

Annals of Mathematics. Ser. 2. 32 : 2, 3.

Anuario (Univ. Nac. de la Plata). 21.

Berichte über die Verhandlungen der Akad. der Wiss. zu Leipzig : Math.-Phys. Klasse.

Boletin Matematico. 3 : 12; **4 :** 1, 2, 3.

Boletin Matematico Elemental.

Boletín del Seminario Matemático Argentino.

Bollettino della Unione Matematica Italiana. 10 : 2, 3.

Bulletin of the American Mathematical Society. 37 : 3, 4, 5, 6.

Bulletin of the Calcutta Mathematical Society. 22 : 4; **23 :** 1.

Contribución al Estudio de las Ciencias Físicas y Matemáticas.

L'Enseignement Mathématique. 29 : 4-6.

Half-Yearly Journal of the Mysore University.

Jahresbericht der Deutschen Mathematiker-Vereinigung. 40 : 5, 6-8, 9-12.

Japanese Journal of Mathematics. 7 : 4; **8 :** 1.

Journal of the Indian Mathematical Society. 19 : 1, 2.

Journal of the London Mathematical Society.

Journal of the Mathematical Association of Japan. 13 : 2, 3.

Journal de la Société Physico-Mathématique de Léningrade.

Mathematical Notes.

Mathematics Teacher.

Memoria (Univ. Nac. de la Plata).

Monatshefte für Mathematik und Physik. 38 : 1.

Nieuw Archief voor Wiskunde. Ser. 2. 17 : 1.

Nieuwe Opgaven.

Periodico di Matematiche. Ser. 4. 11 : 2, 3, 4.

Proceedings of the Edinburgh Mathematical Society. Ser. 2. 2 : 4.

Proceedings of the Physico-Mathematical Society of Japan. Ser. 3. 13 : 2, 3, 4, 5, 6.

Publicaciones de la Facultad de Ciencias Físico-Matemáticas Universidad Nacional de la Plata.

Publications de la Faculté des Sciences de Masaryk.

Quarterly Journal of Mathematics. Oxford Series.

Revista de Ciencias (Peru).

Revista Matemática Hispano-Americana (Madrid). Ser. 2. 6 : 1-2, 3-4, 5-6.

Revue Semestrielle des Publications Mathématiques.

School Science and Mathematics. 31 : 4, 5, 6.

Sitzungsberichte der Berliner Mathematischen Gesellschaft.

Unterrichtsblätter für Mathematik und Naturwissenschaften. 37 : 4, 5, 6, 7, 8.

Wiskundige Opgaven met de Oplossingen. 15 : 1, 2.

LONDON BRANCH—PROGRAMME FOR 1931-32.

President—J. W. N. Sullivan, Esq.

Vice-Presidents—W. C. Fletcher, Esq., C.B.; Sir T. Percy Nunn; Professor Alfred Lodge; Dr. W. F. Sheppard.

Representative on the Council of the Association—J. Katz (Selhurst Grammar).

Chairman—A. W. Siddons (Harrow School).

Vice-Chairmen—Miss L. Zelensky (Haberdashers'), F. C. Boon (Dulwich College.

Hon. Treasurer—Dr. W. G. Bickley (Imperial College), 141 Byrne Road, S.W. 12.

Hon. Secretaries—Miss F. A. Yeldham, "Tralee," Wilmot Way, Banstead, Surrey; C. T. Daltry (Roan School, Greenwich), 112 Canberra Road, S.E. 7.

All meetings will be held at Bedford College, Regent's Park, N.W. 1, on Saturday afternoons at 3. Meetings usually take the form of the reading of a paper, followed by discussion. A summary of each paper is usually posted to members about ten days before the date fixed for its reading. Tea is served after every meeting at 1s. a head.

The purpose of the London Branch is to bring together all who are interested in the teaching of elementary mathematics. Prospective members are welcomed at all meetings, when they may obtain information on the activities of the Branch from either Secretary.

1931.

Oct. 10th. "Arithmetic from the Examiner's Point of View."—B. C. Wallis, Esq., M.A., Chief Examiner to the London County Council.

Nov. 7th. "The First Two Years of Geometry in a Secondary or Preparatory School."—A. W. Siddons, Esq., M.A., Senior Mathematical Master, Harrow School.

Dec. 5th. Presidential Address: "Mathematics and Culture."—J. W. N. Sullivan, Esq.

1932.

Jan. 23rd. Annual Business Meeting, followed by "Some Interesting Practical Problems in Elementary Geometry."—A. Romney Green, Esq. This paper will be illustrated by lantern slides.

Feb. 20th. Meeting for Discussion of Members' Topics. Members are invited to send to either Secretary any topic which they would like to hear discussed at this meeting.

Mar. 19th. "Teaching the Method of Ratio."—F. C. Boon, Esq., B.A., Principal Mathematical Master, Dulwich College.

A Summer Excursion will be arranged in May or June.

FOR SALE.

Mathematical Gazette. 31-211. (Jan. 1902-Jan. 1931) (lacking Nos. 77, 78, 103, 104, 106, 118, 141, 143, 147, 169, 170.) Apply: Mr. L. S. Milward, Stoberry, Malvern, Worcs.

GLEANINGS: AN APPEAL.

The Editor will be grateful for help in the filling up of odd corners. A precise reference should accompany every quotation.

THE MATHEMATICAL ASSOCIATION.

(An Association of Teachers and Students of Elementary Mathematics.)

"I hold every man a debtor to his profession; from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves, by way of amends, to be a help and an ornament thereunto."—BACON (Preface, *Maxims of Law*).

THE MATHEMATICAL ASSOCIATION, which was founded in 1871, as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

Its purpose is to form a strong combination of all persons who are interested in promoting good methods of teaching mathematics. The Association has already been largely successful in this direction. It has become a recognised authority in its own department, and is continuing to exert an important influence on methods of examination.

"The Mathematical Gazette" (published by Messrs. G. BELL & SONS, LTD.) is the organ of the Association. It is issued at least six times a year. The price per copy (to non-members) is usually 2s. 6d. each.

The *Gazette* contains—Articles, Notes, Reviews, etc., dealing with elementary mathematics, and with mathematical topics of general interest.